PROOF OF GOLDBACH,POLIGNAC,LEGENDRE CONJECTURE

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ABSTRACT. In this paper, I suddied about prime number's distribution

1. instruction

Let P_i is i-th prime number

$\bigcup_{i=1}^{k} \bigcup_{n=1}^{\infty} nP_i + a_i$'s longest continual number is smaller than $2P_k$ hence, in the $2P_k$ continual number, it contains P_k -prime number(not divided only by prime numbers that larger than P_k) and when P_k -prime number is smaller than ${P_{k+1}}^2$, it's a prime number

when Composite number $C < P_{k+1} - 1$, there exist at most $\left\lfloor \frac{P_{k+1}-1}{C} \right\rfloor + 1$ numbers in arbitrary P_{k+1} continual numbers when $\sqrt[n]{C} \notin \mathbb{N}$ but $\left\lfloor \frac{P_{k+1}-1}{C} \right\rfloor$ numbers in arbitrary P_{k+1} continual numbers contain more P_k -Composite number (Composite number that divided by P_k or less prime number) than $\{2, 3, \cdots, P_k - 1\}$ but, number of whole factors is same or less, that means arbitrary P_{k+1} continual numbers contain number contain 1 or more P_k -prime number

for example,

2	•	2	•	2	•	2	•	2	•	2
3	•	•	3	•	•	3	•	•	3	•
5	•	•	•	•	5	•	•	•	•	5
7	•	•	•	•	•	•	7	•	•	•

$\{2, 3, \cdots, 11 - 1\}$ contains 7-Composite number

$2 \cdot 3, 2 \cdot 5$ (Don't think about first one) but,

2	•	2	•	2	•	2	•	2	•	2
•	•	3	•	•	3	•	•	3	•	•
5	•	•	•	•	5	•	•	•	•	5
•	•	•	•	7	•	•	•	•	•	•

2000 *Mathematics Subject Classification.* Primary 11P32.
Key words and phrases. Primes, Goldbach's conjecture, Goldbach.

$\{12, 13, \cdots, 22\}$ contains 7-Composite numbers $2 \cdot 3 \cdot n_1, 2 \cdot 5 \cdot n_2$ and $2 \cdot 3 \cdot n_3, 2 \cdot 7 \cdot n_4, 3 \cdot 5 \cdot n_5$

hence, $\bigcup_{i=1}^{k} \bigcup_{n=1}^{\infty} nP_i + a_i$'s longest continual number is smaller than $2P_k$

and $\bigcup \left(\bigcup_{n=1}^{\infty} 2n, \bigcup_{i>1}^{k} \bigcup_{n=1}^{\infty} nP_i + a_i, \bigcup_{i>1}^{k} \bigcup_{n=1}^{\infty} nP_i + b_i \right)$'s longest continual number is

$$\boxed{\begin{array}{ccccc} \bullet & 2 & \bullet & 2 & \bullet \\ \bullet & 3 & \bullet & \bullet & 3 \\ \bullet & \bullet & 5 & \bullet & \bullet \end{array}}' \boxed{\begin{array}{ccccc} \bullet & 2 & \bullet & 2 & \bullet \\ \bullet & 3 & \bullet & \bullet & 3 \\ \bullet & \bullet & 5 & \bullet & \bullet \end{array}} \begin{array}{c} 2 \\ \bullet \\ \bullet \end{array} \boxed{\begin{array}{ccccc} \bullet & 2 & \bullet & 2 & \bullet \\ 3 & \bullet & \bullet & 3 & \bullet \\ \bullet & \bullet & 5 & \bullet & \bullet \end{array}} \boxed{\begin{array}{ccccc} \bullet & 2 & \bullet & 2 & \bullet \\ 3 & \bullet & \bullet & 3 & \bullet \\ \bullet & \bullet & 5 & \bullet & \bullet \end{array}}'$$

it's equal to finding $\begin{array}{cccccccccc} 3 & 3 & \bullet & 3 & 3 & \bullet & 3 & 3 & \bullet & 3 \\ \bullet & 5 & 5 & \bullet & \bullet & \bullet & 5 & 5 & \bullet & \bullet \\ \bullet & \bullet & 7 & 7 & \bullet & \bullet & \bullet & \bullet & \bullet & 7 \\ \bullet & \bullet & \bullet & \bullet & 11 & 11 & \bullet & \bullet & \bullet & \bullet \end{array}$'s lenth $\cdot 2$

but using same method as prook of $\bigcup_{i=1}^{k} \bigcup_{n=1}^{\infty} nP_i + a_i$'s longest continual number is smaller than $2P_k$ it's lenth less than

$$\begin{array}{cccccccccc} 3 & 3 & \bullet & \bullet & \bullet & \bullet & 3 & 3 & \bullet & \bullet \\ \bullet & \bullet & 5 & 5 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & 7 & 7 & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & 11 & 11 \end{array}$$

Let it's smallest prime number P_l that satisfing $P_k < P_l < 2P_k$ we know that

$$\begin{array}{cccccccccc} 3 & 3 & \bullet & \bullet & \bullet & \bullet & 3 & 3 & \bullet & \bullet \\ \bullet & \bullet & 5 & 5 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & 7 & 7 & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & 11 & 11 \end{array}$$

's lenth $\cdot 2 \ < 8P_l < 16p_k$

hence we get

$\bigcup \left(\bigcup_{n=1}^{\infty} 2n, \bigcup_{i>1}^{k} \bigcup_{n=1}^{\infty} nP_i + a_i, \bigcup_{i>1}^{k} \bigcup_{n=1}^{\infty} nP_i + b_i \right)$'s longest continual number is smaller than $16P_k$

2. For any national number k, there are infinitely many pairs such that $p - p' = 2k$

for odd number N, and N+2, N+4, N+6, N+8 isn't a prime number, for example when $N = 113, 211, \cdots$

arrangement like this $\begin{array}{cccc} N^2 & N^2+2 & \cdots & N^2+16N \\ N^2+2k & N^2+2k+2 & \cdots & N^2+16N+2k \end{array}$

$((N+10)^2 = N^2 + 20N + 100 > N^2 + 16N + 2k)$

and we know that there is infinitely many odd number N that N+2, N+4, N+6, N+8 isn't a prime number

and from instruction

we also know $\bigcup\left(\bigcup_{n=1}^{\infty} 2n, \bigcup_{i>1}^{k}\bigcup_{n=1}^{\infty} nP_i + a_i, \bigcup_{i>1}^{k}\bigcup_{n=1}^{\infty} nP_i + b_i\right)$'s longest continual number is smaller than $16P_k$

$(N^2 + 16N) - N^2 + 3 > 16P_m$,

$(N^2 + 16N + 2k) < {P_{m+1}}^2$

$({P_m}^2 \leq N < {P_{m+1}}^2)$

hence, there always exist P_i, P_j that $N^2 < P_i < P_j < (N+10)^2, P_j - P_i = 2k$ $({P_m}^2 \leq N < {P_{m+1}}^2)$ when $N+2, N+4, N+6, N+8$ isn't a prime number and k satisfing $2N + 50 > k$ from $((N+10)^2 = N^2 + 20N + 100 > N^2 + 16N + 2k)$

hence, from there are infinitely many N that N+2,N+4,N+6,N+8 isn't a prime number, For any national number k, there are infinitely many pairs such that $p - p' = 2k$

3. there is a prime number between n^2 and $(n+1)^2$

we can find P_m satisfing

${P_m}^2 < n^2 + 1 < n^2 + 2 < \cdots < (n+1)^2 - 1 < {P_{m+1}}^2$

and $n(n^2+1, n^2+2, \cdots, (n+1)^2 - 1) \geq 2P_m$

hence, from instruction, there is a prime number between n^2 and $(n+1)^2$ for every positive integer n

4. Every even number greater than 2 can be expressed as a sum of two prime numbers

from instruction we also know $\bigcup\left(\bigcup_{n=1}^{\infty} 2n, \bigcup_{i>1}^{k}\bigcup_{n=1}^{\infty} nP_i + a_i, \bigcup_{i>1}^{k}\bigcup_{n=1}^{\infty} nP_i + b_i\right)$'s longest continual number is smaller than $16P_k$

but, in $\begin{matrix} 2N-3 & 2N-5 & \cdots \\ 3 & 5 & \cdots \end{matrix}$ we know that $1, 2, \cdots, 2N$'s largest factor is smaller than $\sqrt{2N}$ hence from $\bigcup\left(\bigcup_{n=1}^{\infty} 2n, \bigcup_{i>1}^{k}\bigcup_{n=1}^{\infty} nP_i + a_i, \bigcup_{i>1}^{k}\bigcup_{n=1}^{\infty} nP_i + b_i\right)$'s longest continual number is smaller than $16P_k$, in continual $16\left\lfloor\sqrt{2N}\right\rfloor$ number's that less than 2N contain prime number. but, when $16\left\lfloor\sqrt{2N}\right\rfloor < N - 1$, Every even number greater than 2 can be expressed as a sum of two prime numbers. but, when $513 < N, 16\left\lfloor\sqrt{2N}\right\rfloor < N - 1$ and we know that when $N \leq 513$,Every even number greater than 2 can be expressed as a sum of two prime numbers hence, Every even number greater than 2 can be expressed as a sum of two prime numbers

5. Sophie Germain conjecture

we know that $\bigcup\left(\bigcup_{n=1}^{\infty} 2n, \bigcup_{i>1}^{k} \bigcup_{n=1}^{\infty} nP_i + a_i, \bigcup_{i>1}^{k} \bigcup_{n=1}^{\infty} nP_i + b_i\right)$'s longest continual number is smaller than $16P_k$

Let

$$f(n) = 2n + 1, f(k) < M < f(k + 2)$$

for odd number k

$$\begin{array}{cccccc} f(k - 16\left\lfloor\sqrt{M}\right\rfloor) & \cdots & f(k-2) & f(k) & < & M \\ k - 16\left\lfloor\sqrt{M}\right\rfloor & \cdots & k-2 & k & < & M \end{array}$$

it contains both $k, f(k)$ are primes when

$$f(k) < M, k < M, 0 < f(k - 16\left\lfloor\sqrt{M}\right\rfloor), 0 < k - 16\left\lfloor\sqrt{M}\right\rfloor$$

but, M, k satisfing

$$f(k) < M, k < M, 0 < f(k - 16\left\lfloor\sqrt{M}\right\rfloor), 0 < k - 16\left\lfloor\sqrt{M}\right\rfloor$$

are infinitely many

Hence,

there are an infinite number of Sophie Germain primes

References

no reference

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