Proof: Our n^{th} proposition is

P_n: "$1+2+\cdots+n=\frac{1}{6}n(n+1)(2n+1)$"

Thus P_1 asserts $1=\frac{1}{6}(1)(1+1)(2+1)$,

P_2 asserts $1+2=\frac{1}{6}(2)(2+1)(4+1)$, ...

In particular, P_1 is a true assertion which serves as our basis for induction.

For the induction step, suppose P_n is true.

That is, we suppose

$$1+2+\cdots+n=\frac{1}{6}n(n+1)(2n+1)$$

is true.

Since we want to prove P_{n+1} from this, we add $n+1$ to both sides to obtain

$$1+2+\cdots+n+(n+1)$$

$$=\frac{1}{6}n(n+1)(2n+1)+(n+1)$$

$$= (n+1)\left[\frac{1}{6}n(2n+1)+1\right]$$
$$= (n+1)\left[\frac{1}{3}n^2+\frac{1}{6}n+1\right]$$
$$= \frac{1}{6}(n+1)(2n^2+n+6)$$

Thus P_{n+1} holds if P_n holds.
By the principle of mathematical induction, we conclude P_n is true for all n.