

This is my attempt to analyze the physics of a standard pushup.

In the discussion below, I will assume that,

- The body is a rigid lever arm.
- The lifting force is applied perpendicular to the plane of the body at the shoulders.
- The hands are directly under the shoulders (not slightly below as in the graphic).

## Human Body Dimensions

It will be helpful to have some average metrics for the human body. I downloaded these from the Internet. The fields highlighted in blue are the raw data. The others are calculated.

<b>Metric</b>	<b>Average Female</b>			<b>Average Male</b>		
Height	1610 mm	63.39 in	63' 4.63"	1740 mm	68.50 in	68' 6.05"
Shoulder height	1310 mm	51.57 in	63' 4.63"	1425 mm	56.10 in	68' 6.05"
Weight	76.4 kg	168.4 lbs	63' 4.63"	88.8 kg	195.8 lbs	68' 6.05"
Shoulder ratio	0.814			0.819		
Center of Mass ratio	0.543			0.560		

Figure 1 Human Body Dimensions

## Standard Pushup

A standard pushup is a second class lever as illustrated in the diagram below. The fulcrum is where the toes touch the floor. The load is the total weight of the entire body applied at the center of mass. The force required to lift the body is what is felt by the muscles in the arms applied at the shoulder joint.

Given,

- $H$  = The height of the person (head to foot).
- $S$  = The height of the person to the shoulders.
- $C$  = The height of the person's center of mass.
- $W$  = The person's total weight.
- $F_s$  = The force, applied at the shoulders, that is needed to lift that weight ( $W$ ).

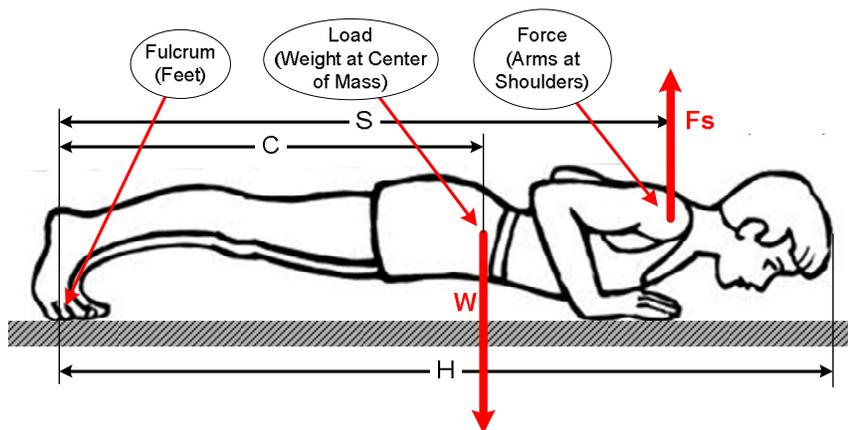


Figure 2 Pushup as Second Class Lever

For any lever in balance, the load times its moment arm equals the force times its moment arm. For the example above,

$$W \times C = F_s \times S \quad (1)$$

To find the force needed to do a pushup, we solve for  $F_s$ .

$$F_s = \frac{W \times C}{S} \quad (2)$$

The force needed to lift the body in a standard pushup is equal to the total body weight times its moment arm (the center of mass as measured from the feet) divided by the height of the shoulders (the moment arm of the force).

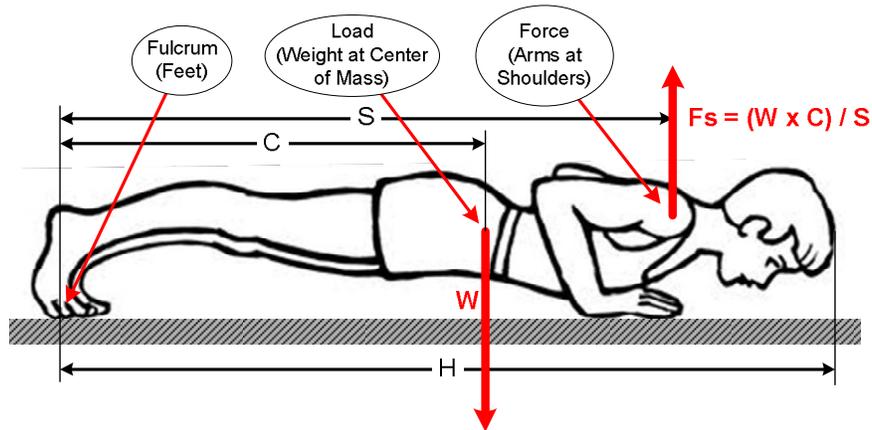


Figure 3 Pushup as Second Class Lever with  $F_s$  Equation

Consider an adult woman, 65" tall (5' 5"), weighing 100 pounds. According to the chart above, her shoulders should be about 52.91" off the ground (65 x 0.814) and her center of mass should be at about 35.295" (65 x .543). For this woman,

$$F_s = \frac{100 \times 35.295}{52.91} = 66.7 \text{ lbs}$$

Because of the mechanical advantage of the lever (the leverage), she does not need to be able to lift her entire weight of 100 pounds. She only has to be able to lift 66.7 pounds in order to do one pushup. But, unlike the bench press, she would also need to be able to keep her body straight and maintain her balance, which is why the pushup is such a good exercise. It involves many more muscles than the bench press, which is more of an isolation exercise.

Using Equation (2), we can calculate the strength needed to do one standard pushup for men or women of various weights.

<b>Weight</b>	<b>Female</b>	<b>Male</b>
100 lbs	66.7 lbs	68.4 lbs
125 lbs	83.4 lbs	85.5 lbs
150 lbs	100.1 lbs	102.6 lbs
175 lbs	116.8 lbs	119.7 lbs
200 lbs	133.5 lbs	136.8 lbs
225 lbs	150.2 lbs	153.9 lbs
250 lbs	166.8 lbs	170.9 lbs

Figure 4 Pushup Strength Requirements

## Inclined Pushup

One simple and effective way to make the pushup easier is to place the hands on an elevated surface. Depending on the elevation, the force required to do one pushup can be reduced to anything from the full force with the hands on the ground to zero with the hands against a wall and standing straight up.

The force is still calculated using the same formula as above, but the weight is reduced by virtue of more of it being directed into the floor.

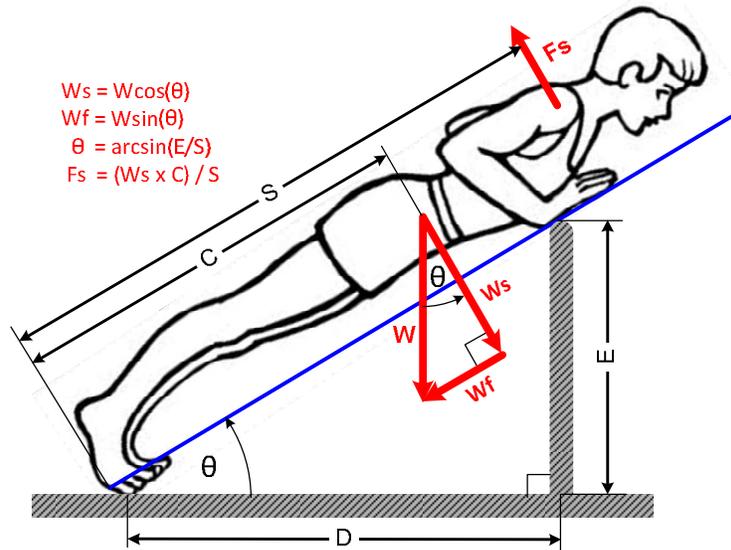


Figure 5 Inclined Pushup Geometry

The total body weight ( $W$ ) is divided into two orthogonal components:  $W_F$  and  $W_S$ .

Component	Formula	Description
$W_F$	$W \times \sin(\theta)$	This is the component that is parallel to the body and is supported by the feet. As the angle gets steeper, more of the weight is supported by the feet and less force is required to lift what remains.
$W_S$	$W \times \cos(\theta)$	This is the component that is perpendicular to the body and is supported by the platform where the hands are resting. This is the component, adjusted for the leverage, that the muscles in the arms must lift. As the angle gets steeper, this component gets smaller and the pushup gets easier.

This illustration shows how these two weight components change as the angle of inclination changes.

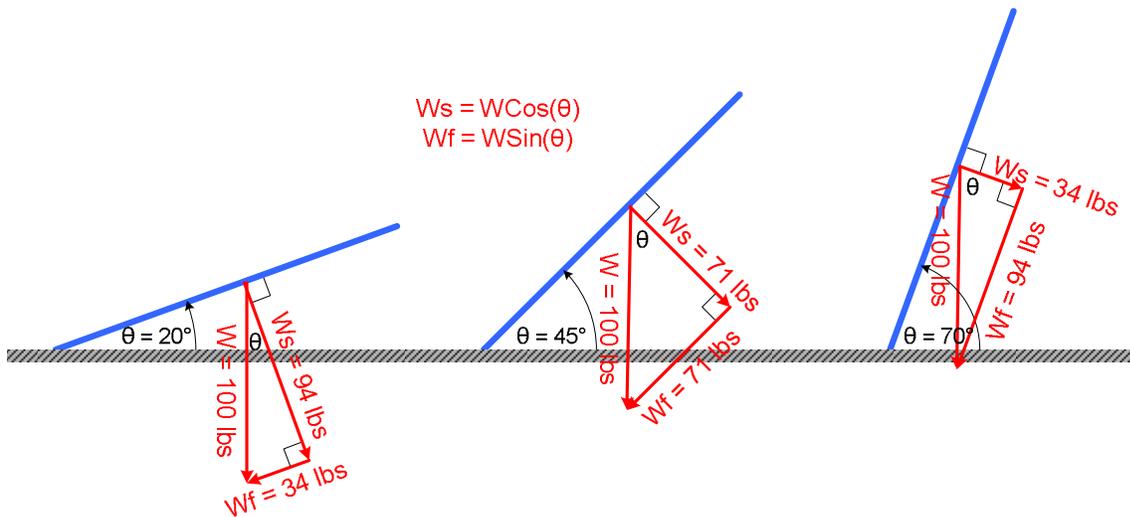


Figure 6 Weight Components

We need to modify equations (1) & (2) above to specify just the component of the weight ( $W$ ) that is perpendicular to the body (parallel to the arms).

$$W_s \times C = F_s \times S \quad (3)$$

$$F_s = \frac{W_s \times C}{S} \quad (4)$$

The next table shows some sample data for the 100-pound adult woman described above. The  $\theta$  column shows the angle of inclination. It goes from  $90^\circ$  (standing pushup, lifting zero weight) to  $0^\circ$  (standard pushup). The  $W_f$  column shows the component of  $W$  that is supported by the feet. It goes from 100 pounds to 0 pounds as the angle goes from  $90^\circ$  to  $0^\circ$ . The  $W_s$  column shows the component of  $W$  that is supported by the shoulders. It goes in the reverse direction, going from 0 pounds to 100 pounds as the angle goes from  $90^\circ$  to  $0^\circ$ . The  $F_s$  column shows the actual force required at the shoulders after applying the mechanical advantage of the lever. And the  $\Delta F_s$  column shows how much weight is added for each change in the angle of inclination.

	<b>100</b>	Weight ( $W$ )					$W_f = W \cdot \sin(\theta)$
	<b>0.814</b>	Average shoulder height parameter ( $K_s$ )					$W_s = W \cdot \cos(\theta)$
	<b>0.543</b>	Average center of mass parameter ( $K_c$ )					$F_s = W_s \times K_c / K_s$
R/C	C	D	E	F	G	H	
	$\theta$	$W_f$	$W_s$	$F_s$	$\Delta F_s$	<b>Comments</b>	
7	$0^\circ$	0.0	100.0	66.7	--	Standard pushup. Shoulders support full pushup weight.	
8	$10^\circ$	17.4	98.5	65.7	-1.0		
9	$20^\circ$	34.2	94.0	62.7	-3.0		
10	$30^\circ$	50.0	86.6	57.8	-4.9	Feet support half of the weight.	
11	$40^\circ$	64.3	76.6	51.1	-6.7		
12	$45^\circ$	70.7	70.7	47.2	-3.9	Feet and shoulders support equal weight.	
13	$50^\circ$	76.6	64.3	42.9	-4.3		
14	$60^\circ$	86.6	50.0	33.4	-9.5	Shoulders support half of the weight.	
15	$70^\circ$	94.0	34.2	22.8	-10.5		
16	$80^\circ$	98.5	17.4	11.6	-11.2		
17	$90^\circ$	100.0	0.0	0.0	-11.6	Standing "pushup". Shoulders support zero weight.	

Figure 7 Sample Data (Based on Weight)

Row 8 ( $0^\circ$ ) is a standard pushup. We get the same result (67) as above. If we increase the angle slightly to  $10^\circ$ ,  $W_s$  drops to 98 and  $F_s$  to 66. It's a small inclination angle, so the advantage is also small.

At an angle of  $30^\circ$  (row 11),  $W_F$  increases to 50 pounds, half the weight.  $W_S$  and  $F_S$  decrease to 87 pounds 58 pounds respectively.

At an angle of  $45^\circ$  (row 13), half way up,  $W_F$  and  $W_S$  are equal and  $F_S$  is down to 47 pounds.

At an angle of  $60^\circ$  (row 15),  $W_F$  increases to 87 pounds and  $W_S$  drops to 50 pounds, half the weight. This is the reverse of the values at  $30^\circ$ .  $F_S$  decreases to 33, half of the force at  $0^\circ$ .

At an angle of  $80^\circ$  (row 17),  $W_F$  increases to 98 pounds. Almost all of the weight is being supported by the feet.  $W_S$  drops to 17 and  $F_S$  to 12. I would think that almost anyone could lift 12 pounds. From there, the angle can be gradually reduced, which will gradually increase the weight as the muscles and joints get used to the work.

Row 18, at an angle of  $90^\circ$ , is for a standing “pushup”. It’s really not a pushup at all as there is zero weight to lift. All of it is being supported by the feet.

These calculations require three parameters: weight ( $W$ ), shoulder height ( $S$ ), and center of mass ( $C$ ). Most people know their weight, but few people know the other two parameters. Next we will examine ways to obtain them.