$\boxed{35}$ $x^2y'' - 4xy' + 6y = x^4\sin(x)$ ① $\quad f(x) = x^4\sin(x)$

$y_{homogeneous} = c_1y_1 + c_2y_2$ replace c_1 and c_2 by $v_1(x)$ and $v_2(x)$

determine ~~homogeneous~~ homogeneous equation

$y_p(x) = v_1(x)y_1(x) + v_2(x)y_2(x)$ $\quad y_1$ and y_2 are known functions

~~y =~~ Need to ~~solve~~ find $v_1(x)$ and $v_2(x)$

$$v_1'y_1 + v_2'y_2 = 0$$
$$v_1'y_1' + v_2'y_2' = f(x)$$

~~$v_1 = \int v_1$~~

$$y_p'(x) = v_1'(x)y_1(x) + v_1(x)y_1'(x) + v_2'(x)y_2(x) + v_2(x)y_2'(x)$$

remembering $v_1'(x)y_1(x) + v_2'(x)y_2(x) = 0$

$$y_p'(x) = v_1(x)y_1'(x) + v_2(x)y_2'(x)$$

$$y_p''(x) = v_1'(x)y_1'(x) + v_1(x)y_1''(x) + v_2'(x)y_2'(x) + v_2(x)y_2''(x)$$

~~Rewrite eqn ① as~~ Using $y_p''(x)$, $y_p'(x)$ and $y_p(x)$ in ① we get

$$x^2\left[v_1'(x)y_1'(x) + v_1(x)y_1''(x) + v_2'(x)y_2'(x) + v_2(x)y_2''(x)\right] - 4x\left[v_1(x)y_1'(x) + v_2(x)y_2'(x)\right] + 6\left[v_1(x)y_1(x) + v_2(x)y_2(x)\right] = x^4\sin(x)$$

~~eq~~ rearranging equation

$$v_1(x)\left[x^2y_1''(x) - 4xy_1'(x) + 6y_1(x)\right] + v_2(x)\left[x^2y_2''(x) - 4xy_2'(x) + 6y_2(x)\right] + x^2\left[v_1'(x)y_1'(x) + v_2'(x)y_2'(x)\right] = x^4\sin(x)$$