

$$x^2 [V_1'(x) y_1'(x) + V_2'(x) y_2'(x)] = x^4 \sin(x)$$

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$$V_1'(x) y_1(x) + V_2'(x) y_2(x) = 0$$

$$V_1'(x) y_1(x) = -V_2'(x) y_2(x)$$

$$V_1'(x) y_1'(x) + V_2'(x) y_2'(x) = x^2 \sin(x)$$

Let $A = V_1'(x)$ and $B = V_2'(x)$

$$\frac{d}{dx} \left[\frac{\sin(x)}{\cos(x)} \right] = \frac{\cos(x)\cos(x) - [\sin(x)](-\sin(x))}{\cos^2(x)} \Rightarrow A y_1'(x) + B y_2'(x) dx = \int x^2 \sin(x)$$

$$\frac{d}{dx} [\cos(x)] = -\sin(x)$$

$$\int x^2 \sin(x) dx = -x^2 \cos(x) - \int 2x [-\cos(x)] dx = -x^2 \cos(x) + \int 2x \cos(x) dx$$

$$= -x^2 \cos(x) + [2x \sin(x)] - \int 2 \sin(x) dx$$

$$= -x^2 \cos(x) + [2x \sin(x) + 2 \cos(x)]$$

$$\text{Ans} = y = Ax^2 + Bx^3 - x^2 \sin(x)$$

[Faint handwritten notes and scribbles at the bottom of the page]