$$x^2\left[v_1'(x)\,y_1'(x) + v_2'(x)\,y_2'(x)\right] = x^4\sin(x)$$

$$v_1'(x)\,y_1(x) + v_2'(x)\,y_2(x) = 0$$

$$v_1'(x)\,y_1(x) = -v_2'(x)\,y_2(x)$$

$$v_1'(x)\,y_1'(x) + v_2'(x)\,y_2'(x) = x^2\sin(x)$$

Let $A = v_1'(x)$ and $B = v_2'(x)$

$$\int Ay_1'(x) + By_2'(x)\,dx = \int x^2\sin(x)$$

$\frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right]$

$= \frac{\cos(x)\cos(x) - [-\sin(x)]\sin(x)}{\cos^2(x)}$

$\frac{d}{dx}[\cos(x)]$

$= -\sin(x)$

$$\int x^2\sin(x)\,dx = -x^2\cos(x) - \int 2x[-\cos(x)]\,dx$$

$$= -x^2\cos(x) + \int 2x\cos(x)\,dx$$

$$= -x^2\cos(x) + \left[[2x\sin(x)] - \int 2\sin(x)\,dx\right]$$

$$= -x^2\cos(x) + [2x\sin(x) + 2\cos(x)]$$

$$\text{Ans: } y = Ax^2 + Bx^3 - x^2\sin(x)$$