M=10kg-mass

C=100Ns/m-damping ceofficent

K=1000N/m spring stiffnes

Initial condtions t0=0,x0=0.04m,x'0=0(velocity at t=t0), x''0=0(acceleration at t=t0)

Solution

$$ma = -kx - cV$$

$$mx'' = -kx - cx'$$

$$x'' + \frac{c}{m}x' + \frac{k}{m}x = 0$$

Putting known values into equation:

$$x'' + \frac{100}{10}x' + \frac{1000}{10}x = 0$$

$$x'' + 10x' + 100x = 0$$

Now solving using power series:

Let assume that:

$$x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + ..$$

$$t_0 = 0, x_0 = 0.04 \; so \; a_0 = 0.04 \;\; From \; initial \; conditions$$

$$x'(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + ..$$

$$t_0 = 0, x'_0 = 0 \; so \; a_1 = 0 \;\; From \; initial \; conditions$$

$$x''(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3 + ..$$

$$t_0 = 0, x''_0 = 0 \; so \; a_2 = 0 \;\; From \; initial \; conditions$$

Putting these into differential equation

$$x'' + 10x' + 100x = 0$$

$$(2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3 + ..) + 10(a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + ..) +$$

$$+100(a_0 + a_1 t + a_2 t^2 + a_3 t^3 + ..) = 0$$

Rearranging

$$100a_0 + 10a_1 + 2a_2 + (100a_1 + 20a_2 + 6a_3)t + (100a_2 + 30a_3 + 12a_4)t^2 +$$

$$+(100a_3 + 40a_4 + 20a_5)t^3 = 0$$

And $a_0 = 0.04$, $a_1 = 0, a_2 = 0$ $-$ *From initial conditions*

$$0.04 + (100a_1 + 20a_2 + 6a_3)t + (100a_2 + 30a_3 + 12a_4)t^2 + (100a_3 + 40a_4 + 20a_5)t^3 = 0$$