

M=10kg-mass

C=100Ns/m-damping coefficient

K=1000N/m spring stiffness

Initial conditions $t_0=0, x_0=0.04\text{m}, x'_0=0$ (velocity at $t=t_0$),

Solution

$$ma = -kx - cV$$

$$mx'' = -kx - cx'$$

$$x'' + \frac{c}{m}x' + \frac{k}{m}x = 0$$

Putting known values into equation:

$$x'' + \frac{100}{10}x' + \frac{1000}{10}x = 0$$

$$x'' + 10x' + 100x = 0$$

Now solving using power series:

Let assume that:

$$x(t) = a_0 + a_1t + a_2t^2 + a_3t^3 + \dots$$

$$t_0 = 0, x_0 = 0.04 \text{ so } a_0 = 0.04 \text{ From initial conditions}$$

$$x'(t) = a_1 + 2a_2t + 3a_3t^2 + 4a_4t^3 + \dots$$

$$t_0 = 0, x'_0 = 0 \text{ so } a_1 = 0 \text{ From initial conditions}$$

$$x''(t) = 2a_2 + 6a_3t + 12a_4t^2 + 20a_5t^3 + \dots$$

Putting these into differential equation

$$x'' + 10x' + 100x = 0$$

$$(2a_2 + 6a_3t + 12a_4t^2 + 20a_5t^3 + \dots) + 10(a_1 + 2a_2t + 3a_3t^2 + 4a_4t^3 + \dots) + 100(a_0 + a_1t + a_2t^2 + a_3t^3 + \dots) = 0$$

Rearranging

$$100a_0 + 10a_1 + 2a_2 + (100a_1 + 20a_2 + 6a_3)t + (100a_2 + 30a_3 + 12a_4)t^2 + (100a_3 + 40a_4 + 20a_5)t^3 = 0$$

And $a_0 = 0.04$, $a_1 = 0$ – From initial conditions

Comparing coefficients

$$100a_0 + 10a_1 + 2a_2 = 0$$

$$100 * 0.04 + 10 * 0 + 2a_2 = 0$$

$$a_2 = -2$$

$$100a_1 + 20a_2 + 6a_3 = 0$$

$$100(0) + 20(-2) + 6a_3 = 0$$

$$a_3 = \frac{40}{6}$$

$$100a_2 + 30a_3 + 12a_4 = 0$$

$$100(-2) + 30\left(\frac{40}{6}\right) + 12a_4 = 0$$

$$a_4 = 0$$

$$100a_3 + 40a_4 + 20a_5 = 0$$

$$100\left(\frac{40}{6}\right) + 40(0) + 20a_5 = 0$$

$$20a_5 = -\frac{200}{3}$$

Finally variation of displacement with time will be described by equation bellow:

$$x(t) = a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 + a_5t^5 + ..$$

$$x(t) = 0.04 - 2t^2 + \frac{40}{6}t^3 + \frac{200}{6}t^5 + ..$$

