A vehicle travels the first half of its journey with a constant velocity v_1. It then travels the second half of the journey at a constant velocity v_2. (i) What is its average velocity for the whole journey? (ii) If it wants to have an average velocity of $2v_1$, what would v_2 have to be?

(i) One has to start with the definition of "average velocity". Here, it is defined as the total displacement divided by total time taken. If we let the length of the total journey be l, and the total time t, then the average velocity is

$$\langle v \rangle = l/t \,. \qquad (1)$$

Each half of the journey covers a distance of $l/2$. Thus

$$v_1 t_1 = l/2 \qquad \text{and} \qquad v_2 t_2 = l/2$$

where t_1 and t_2 are the time taken over each half of the journey. Since $t = t_1 + t_2$, substituting this into Eq. 1 give us

$$\langle v \rangle = \frac{l}{\frac{l}{2v_1} + \frac{l}{2v_2}} \,.$$

Simplifying, we get

$$\langle v \rangle = \frac{2v_1 v_2}{v_1 + v_2} \,. \qquad (2)$$

Notice that this is *not* $(v_1 + v_2)/2$, as one would expect from the average of two numbers.

(ii) We first use the result from Part (i) and find the expression for v_2. Rearranging Eq. 2, we obtain

$$v_2 = \frac{v_1 \langle v \rangle}{2v_1 - \langle v \rangle} \,.$$

What would v_2 have to be if we want $\langle v \rangle = 2v_1$? One can immediately see from the denominator that one would encounter a problem. It implies that v_2 has to be infinitely large for the average velocity to be $2v_1$! Is this a reasonable result?

The problem here is that as v_2 increases, it is spending less time travelling the given distance. The average velocity does not only take into account how far an object travels with that velocity, but also how long of a time it took. The average

velocity is “weighted” for both distance and time. So as v_2 increase, it is also taking less time to travel a distance of $l/2$. Thus, it can never travel fast enough and spend enough length of time to get the average velocity to $2v_1$.