Quantum Foundations I: What is it all about & (why) should mathematical physicists care?

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Quantum foundations = general principles of quantum theory

- Quantum foundations have been set
in the 20's and 30's of the 20th century.
- Yet some physicists still work on it. Why?
- Because they **question** those foundations
and try to **reformulate** them.

- Is there an experiment suggesting that something about
the old foundations is wrong? No.
- Quantum theory still works perfectly well in practice.

- So what's the point of questioning its foundations?

Analogy in mathematics:

- The foundations of calculus have been set by Newton and Leibniz.
- It works perfectly well in practice
(theoretical physics, engineering, ...).

- Yet Cauchy, Weierstrass and others reformulated calculus
in terms of ε and δ.
(For every $\varepsilon > 0$ there is a $\delta > 0$ such that ...)

- What's the point of reformulating something that works
perfectly well in practice?

- The point is: internal **logical consistency** and **clarity**!!!

- In Leibniz caclulus, the "infinitesimal" dx
sometimes treated as $dx > 0$, sometimes as $dx = 0$.
- Logically inconsistent, or at least not clear
when is $dx > 0$ and when is $dx = 0$.
- In practice one decides by intuition.
- The idea of ε-δ reformulation is to replace intuition
with something more precise.

The form of an ideal mathematical theory:

1. Primitive objects - understood intuitively, not defined formally.

- It is impossible to define the meaning of **every** word.
(Because any definition is an array of other words,
and one doesn't want infinite regress or circularity.)
- Example in set theory: the general notion of a "set".

2. Definitions - objects (words) with formally defined meaning.

3. Axioms - statements (about the objects) which are not proved.

- Axioms usually chosen so that they look intuitively obvious.
- Example in set theory: Zermelo-Fraenkel + axiom of choice

4. Theorems - statements proved from the axioms.

Standard axioms of quantum theory (sketch):

1. The unitary evolution axiom:

$$\widehat{H}|\psi(t)\rangle = i\frac{\partial|\psi(t)\rangle}{\partial t}$$

2. The measurement axiom:
When the observable

$$\hat{A} = \sum_a a|a\rangle\langle a|$$

is measured, then $|\psi\rangle$ "collapses" to one of the states in the set $\{|a\rangle\}$

$$|\psi\rangle \to |a\rangle$$

3. The Born rule axiom:
The probability of the "collapse" is

$$p_a = |\langle a|\psi\rangle|^2$$

- In practice, any quantum phenomenon can be thought of as some realization of those 3 axioms.
- Different quantum systems differ only by what exactly is $\widehat{H}$ and what exactly is $\hat{A}$.

- The most problematic is Axiom 2 - the measuremet axiom:
"When observable is **measured**, then ..."
- What does it mean "measured"?
- How is "measurement" defined?

- There is **no definition** of "measurement" in standard formulation of quantum theory.
- "Measurement" is a **primitive** notion.
- Not every interaction counts as measurement, so how do we know which interaction is measurement and which isn't?
- In practice, we decide by physical intuition.

- Sure, **some** notions must be taken as primitive.
- But **measurement**?
- Is it really the case that measurement cannot be reduced to something more elementary?
- Or if it can, what exactly it is?

- This is **the measurement problem** of quantum theory, the central problem in quantum foundations.

The analogy with calculus:

- What is measurement?
- What kind of physical process is that?

Analogous to:
- What is the infinitesimal dx?
- What kind of number is that?

Apparent logical contradiction, or at least vagueness:
- $|\psi(t)\rangle$ sometimes evolves according to Axiom 1 (unitary evolution) and sometimes according to Axiom 2 (collapse).

Analogous to:
- dx is sometimes $dx > 0$ and sometimes $dx = 0$.

- Standard solution for calculus: ε-δ formulation.
- Alternative solution (Robinson 1961): nonstandard analysis (mathematically precise definition of infinitesimals).

- Is there an analogous solution of the measurement problem?
- That's what research in quantum foundations is all about.
- So far there is no universally accepted solution.

Some of the proposed solutions:

"Copenhagen" shut up and calculate:
- It may be acceptable for practical physicists, but perhaps not for mathematical physicists who seek rigor and logical precision.

Bohr "Copenhagen" interpretation:
- Measurement is macroscopic and hence described by classical (not quantum) physics.

Problems:
1. Where exactly is the borderline between macro and micro?
2. Quantum cannot be formulated without referring to classical, so one cannot say that quantum is more fundamental than classical.

Von Neumann "Copenhagen" interpretation:
- Collapse is somehow caused by consciousness.
Problems:
1. Why is consciousness important for the quantum (micro) but not for the classical (macro)?
2. Is consciousness primitive, or can it be reduced to somethig else? To what?
3. What about physics before biological life as we know it?
4. Many other philosophical questions ...

QBism (modern "Copenhagen"):
- It's all about **information**.
- Physics is **not** about the real world out there.
- Physics is about **our** (incomplete) information about the world.
- Collapse is just our subjective update of information.
Problems:
- Similar to the problems with consciousness.

Ontological approaches:

Physics is about the real world out there.
Fundamental axioms do not refer to "measurement".
General questions:
- Kinematics: What the real world is made of?
- Dynamics: How its properties change with time?

Many worlds:
- Kinematics: $|\psi(t)\rangle$
- Dynamics: Schrödinger equation $\widehat{H}|\psi(t)\rangle = i\partial_t|\psi(t)\rangle$

$\Rightarrow$ No collapse, all possible outcomes are realised.
Problem:
- Where does the Born rule (probability) come from?

Objective collapse theories:
- Kinematics: $|\psi(t)\rangle$
- Dynamics: Modified Schrödinger equation with an additional nonlinear stochastic term responsible for the collapse.

Problem:
- Looks very *ad hoc*, additional term not unique.

Bohmian mechanics:
- Kinematics: the world is made of pointlike particles
with trajectories $\mathbf{X}_a(t)$.
- Dynamics: deterministic, the velocities $\dot{\mathbf{X}}_a(t)$ of n particles,
$a = 1, \dots n$, determined by the wave function $\psi(\mathbf{x}_1, \cdots, \mathbf{x}_n, t)$
through the formula

$$\dot{\mathbf{X}}_a = \frac{-i\hbar}{2m_a} \frac{\psi^* \overleftrightarrow{\boldsymbol{\nabla}}_a \psi}{\psi^* \psi}$$

- Probability (Born rule) emerges
in a way similar to classical statistical mechanics.
Problem:
- How to generalize to relativistic QFT?
(I worked on that problem a lot.)

Frequent arguments against working on quantum interpretations/foundations:

1. It's not science because it doesn't make measurable predictions.

Reply: It does make measurable predictions, usually the same ones as the standard textbook quantum theory.

2. It's useless because it rarely makes **new** measurable predictions.

Reply: The axiomatic/contructive QFT (mathematically rigorous QFT), for instance, also doesn't make new measurable predictions,
yet mathematical physicists don't object that it's useless.
- The primary motivation is to get deeper understanding,
new predictions may arise as a side effect.

3. Experts can't agree on which interpretation is right.

Reply: Experts also can't agree, for instance, what's the right approach to quantize gravity (strings, loop quantum gravity, ...)
yet mathematical physicists don't use it as an argument against working on quantum gravity.

4. Questions in quantum foundations are not formulated in a clear mathematical form.

Reply:
- Some of them are.
- Some of them were not in the past, but they are now.
- Some are still not, but that's exactly what many people in the field are working on.

How mathematical physics contributes to quantum foundations?

- As to the rest of theoretical physics, by making it mathematically more precise.

No-go theorems:
- Prove how quantum theory can **not** be interpreted.
- Similarly to the Gödel incompleteness theorems,
they are mathematical but have deep philosophical consequences.

1. Kochen-Specker theorem (1967):
- proof of contextuality

Suppose some observables (e.g. the value od spin in all directions) have some values before one measures them.
$\Rightarrow$ Measurement necessarily **changes** that values.

Interpretation ruled out:
- measurement reveales values that existed before the measurement.

Interpretations consistent with the theorem:
a) values before measurement don't exist (Copenhagen), or
b) they exist but measurement changes them.

2. Bell theorem (1964):
- proof of nonlocality

Suppose some variables (e.g. particle positions) have values before one measures them.
$\Rightarrow$ Measurement changes that values by a **nonlocal** law.

Interpretation ruled out:
- the evolution of values is governed by a local law.

Interpretations consistent with the theorem:
a) values before measurement don't exist (Copenhagen), or
b) they exist but measurement changes them by a nonlocal law.

Implicit asumptions in the theorem that are sometimes questioned:
- classical logic
- single outcomes (no many worlds)
- no backward causation (no influence from the future)
- free choice (choosing which observable will be measured not correlated with the state of the measured system or with choice of observable to be measured in another subsystem)

3. Pusey-Barrett-Rudolph (PBR) theorem (2012):

- proof that wave function is "real"

Suppose nature is completely described by a set of variables
that have some values even when nobody measures them.
$\Rightarrow$ The quantum state $|\psi\rangle$ can be reproduced
from those values.

Interpretation ruled out:
- $|\psi\rangle$ is just a Bayesian probability representing
our incomplete knowledge about the real world and nothing more.
- If we knew the real world itself,
from that alone we could not reproduce $|\psi\rangle$.

Interpretations consistent with the theorem:
a) there is no real world out there,
there are only our observations (radical Copenhagen), or
b) there is a real world out there and $|\psi\rangle$ is a part of it.

Quantum theory of measurement:

Measurement problem - the central problem in quantum foundations.
- A lot can be said without assuming any particular interpretation.

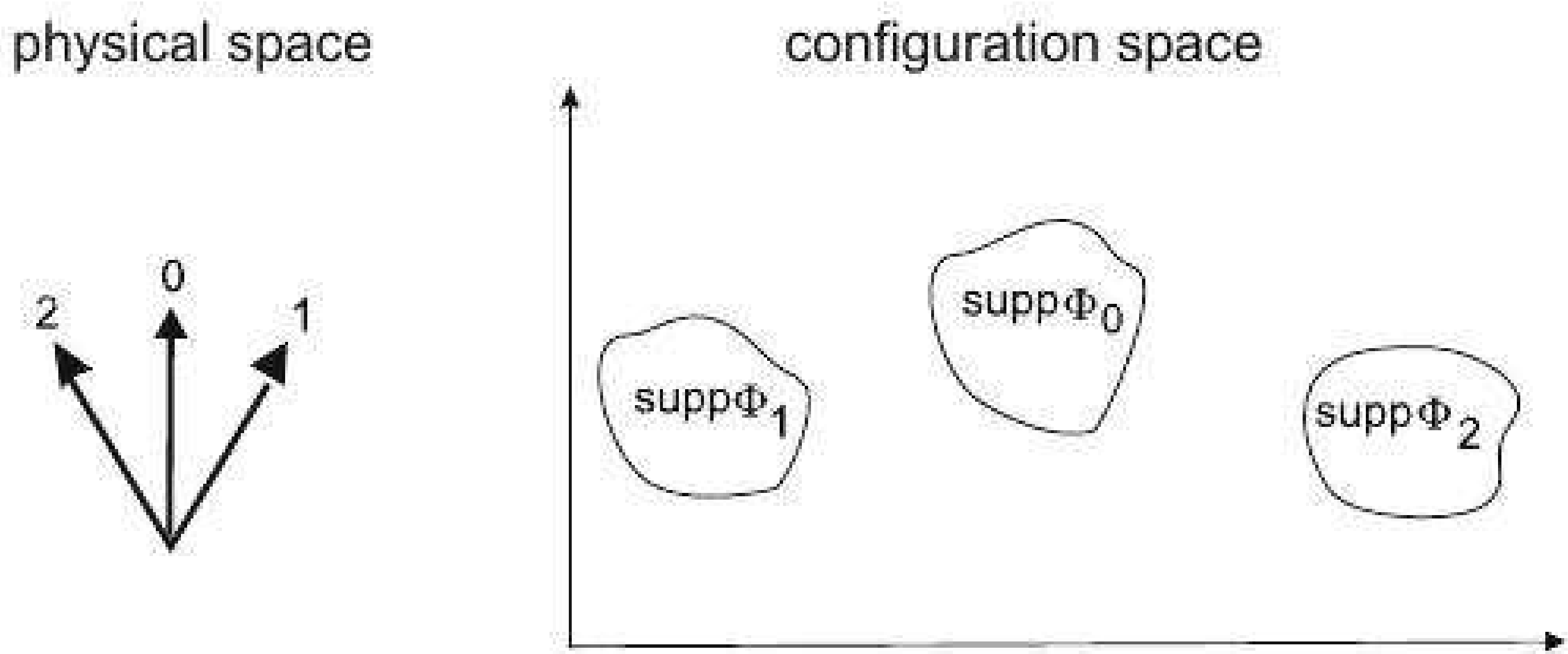
Main idea:
- Treat the macroscopic measuring apparatus as a **quantum** object.
$\Rightarrow$ The measuring apparatus has a wave function!

Measure observable $\hat{K}$ with eigenstates $|k\rangle$.
$|k\rangle$ - states of the measured quantum object
(electron, photon, ...)

Measuring apparatus in initial state $|\Phi_0\rangle$.
Interaction $\Rightarrow$ unitary continuous transition

$$|k\rangle|\Phi_0\rangle \to |k'\rangle|\Phi_k\rangle$$

$|\Phi_0\rangle$ and $|\Phi_k\rangle$ are **macroscopically distinguishable** pointer states.
$\Rightarrow$ Wave functions have a negligible overlap in **configuration space**.



$$\Phi_{k_1}(\vec{x})\Phi_{k_2}(\vec{x}) \simeq 0 \quad \text{for} \quad k_1 \neq k_2$$

where $\Phi_k(\vec{x}) \equiv \langle \vec{x}|\Phi_k\rangle$, $\vec{x} \equiv (\mathbf{x}_1, \ldots, \mathbf{x}_n)$,
n = number of particles constituting the apparatus

For a superposition $|\psi\rangle = \sum_k c_k |k\rangle$:

$$\boxed{|\psi\rangle|\Phi_0\rangle \to \sum_k c_k |k'\rangle|\Phi_k\rangle}$$

Why this "collapses" to $|k'\rangle|\Phi_k\rangle$?

$|\Phi_k\rangle$ are macroscopically distinguishable.
$\Rightarrow$ Superposition consists of many distinguishable branches.
Each branch evolves as if other branches did not exist.
$\Rightarrow$ From perspective of any branch, other branches do not exist.
Explains the collapse if one remaining question can be answered:

Why should we take a view from the perspective of a branch as the physical one?

- Different interpretations offer different answers.
- Not to be discussed today.

Possible topics for next talks:

Quantum Foundations II: Some no-go theorems

Quantum Foundations III: Bohmian mechanics and instrumentalism

Quantum Foundations IV: Bohmian mechanics and relativistic QFT

Quantum Foundations V: Suggestions welcome