Quantum optical tests of complementarity

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Simultaneous observation of wave and particle behaviour is prohibited, usually by the position-momentum uncertainty relation. New detectors, constructed with the aid of modern quantum optics, provide a way around this obstacle in atom interferometers, and allow the investigation of other mechanisms that enforce complementarity.

COMPLEMENTARITY distinguishes the world of quantum phenomena from the realm of classical physics. The lion's share of the credit for teaching us to accept complementarity as a fact and for insisting that we have to learn to live with it belongs to Niels Bohr. In 1927, when he was reviewing the subject at Como[1] in a speech delivered in honour of Count Alessandro Volta (1745–1827), quantum theory as we know it today was still new, and all examples used to illustrate complementarity referred to the position (particle-like) and momentum (wave-like) attributes of a quantum mechanical object, be it a photon or a massive particle. This is the historical reason why complementarity is often superficially identified with the 'wave-particle duality of matter'.

Richard Feynman, discussing the two-slit experiment in his admirable introduction to quantum mechanics[2], notes that this wave-particle dual behaviour contains the basic mystery of quantum mechanics. In fact, he goes so far as to say: "In reality *it* contains the only mystery."

Complementarity, however, is a more general concept. We say that two observables are 'complementary' if precise knowledge of one of them implies that all possible outcomes of measuring the other one are equally probable. We may illustrate this by two extreme examples. (A more general discussion is given in ref. 3.) The first example consists of the position and momentum (along one direction) of a particle: if, say, the position is predetermined then the result of a momentum measurement cannot be predicted and all momentum values are equally probable (in a large range). The second extreme involves two orthogonal spin components of a spin-1/2 particle: if, say, the vertical spin component has a definite value ('up' or 'down') then upon measuring a horizontal component both values ('left' or 'right', for instance) are found, each with a probability of 50%.

Here then is the 'Principle of Complementarity':

> For each degree of freedom the dynamical variables are a pair of complementary observables.

A less formal, less precise version in practical terms is:

> No matter how the system is prepared, there is always a measurement whose outcome is utterly unpredictable.

Thus, in the microcosmos complete knowledge of the future in the sense of classical physics is not available. This, in essence, is the 'mystery' pointed to by Feynman.

As is true for all physical principles, the actual mechanisms that enforce complementarity vary from one experimental situation to another. Over the years various gedanken experiments have been analysed which emphasize this complementarity in quantum mechanics. Examples include Albert Einstein's recoiling-slit arrangement[4] (analysed in the spirit of Willis Lamb[5] in ref. 6), Feynman's electron-light scattering scheme[2] and Werner Heisenberg's microscope[7]. In the first two of these examples Heisenberg's position-momentum uncertainty relation[8]

$$\delta x \delta p \geqslant \frac{\hbar}{2} \qquad (1)$$

makes it impossible to determine which hole the electron (or photon) passes through without at the same time disturbing the electrons (photons) enough to destroy the interference pattern. Similar conclusions are reached by Heisenberg in his classic microscope example.[7] In the present work we have found a way around this position-momentum uncertainty obstacle.

That is, we have found a way, based on matter-wave interferometry, and recent advances in quantum optics, namely the micromaser[9-14] and laser cooling[15,16], to obtain which-path or particle-like information without scattering or otherwise introducing large uncontrolled phase factors into the interfering beams. To be sure, we find that the interference fringes disappear once we have which-path information, but we conclude that this disappearance originates in correlations between the measuring apparatus and the systems being observed. The principle of complementarity is manifest although the position-momentum uncertainty relation plays no role.

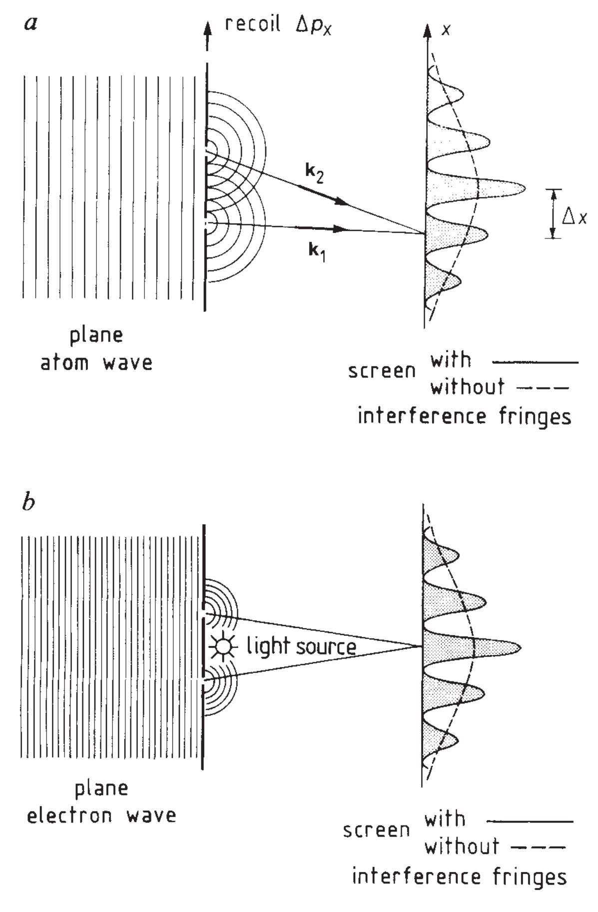

FIG. 1 *a*, Einstein's variant of the two-slit experiment. In this gedanken experiment the slits can recoil and reveal through which slit the photon reached the screen, inasmuch as only one of the wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ is consistent with a known amount of recoil momentum. *b*, Feynman's version of Einstein's gedanken experiment. Here electrons interfere, and the scattering of photons is used to detect their position just behind the slits, revealing through which slit the electron reached the screen.

Gedanken experiments illustrating complementarity

We now turn to a brief survey of the usual textbook examples. (As a recent textbook we recommend ref. 17.) These examples traditionally involve a two-slit experiment in which light is allowed to interfere on a distant screen, thereby showing interference phenomena, the hallmark of wave-like behaviour. However, if we are able to detect which path the light has followed, we have particle-like information, and Nature refuses to let us observe wave-like phenomena.

Perhaps the archetypal example is Einstein's recoiling-slit arrangement[4,6], depicted in Fig. 1*a*. Einstein hoped, by this example, to give a gedanken experiment which would yield both which-path (German: *welcher Weg*) information and also show wave-like interference phenomena. But Bohr[4] pointed out that we must also treat the recoiling slits by the laws of quantum mechanics. As discussed in Box 1, one then learns that there cannot be an interference pattern if the experiment allows us to determine through which slit the photon reached the screen.

In another example along these lines Feynman[2] replaces the photons by electrons. As the wave nature of matter is well known, interference between the electrons passing through slits, as in Fig. 1*b*, would be expected to lead to the usual fringe pattern on the screen. (Indeed, precision experiments, in which slow neutrons with a de Broglie wavelength of ~20 Å pass through two macroscopic slits with widths of ~150 μm demonstrate perfect agreement with the quantum mechanical predictions; see Fig. 2.) In this scheme we now have an extra 'handle' on the interfering particles as electrons can be observed by, for example, light scattering. This is depicted in Fig. 1*b* where we see a light source which would scatter light from the vicinity of either slit depending on which slit the electron comes through. Feynman then goes on to explain that this observation procedure destroys the interference pattern as seen on the screen. He concludes his analysis of this interesting example with the following statement:

> If an apparatus is capable of determining which hole the electron goes through, it *cannot* be so delicate that it does not disturb the pattern in an essential way. No one has ever found (or even thought of) a way around the uncertainty principle.

In the experimental situations discussed so far, as in all standard examples, including Heisenberg's famous microscope[7], complementarity is enforced with the aid of Heisenberg's position-momentum uncertainty relation. Is this mechanism always at work? No! We have recently[18-20] found a way around it.

We have developed and analysed a scheme that is very much in the spirit of Einstein's original proposal: we observe which path the particle has followed and do this without appreciably altering the spatial wave function. If we can do this, we will have shown that Einstein's goal is realizable, and the question of how the principle of complementarity is enforced must then be readdressed. Here we will show that Einstein's goal is indeed obtainable: it is possible to obtain *welcher Weg* information without exposing the interfering beams to uncontrollable scattering events.

On the other hand, Bohr would not have been distressed by the outcome of these considerations, as wave-like (interference) phenomenon is lost as soon as one is able to tell which path the atom traversed. Quantum mechanics contains a built-in safeguard such that the loss of coherence in measurements on quantum systems can always be traced to correlations between the measuring apparatus and the system being observed. That is to say, in the present example, it is simply the information contained in a functioning measuring apparatus that changes the outcome of the experiment, and not uncontrollable alterations of the spatial wave function, resulting from the action of the measuring apparatus on the system under observation.

These considerations are based largely on recent advances in the field of quantum optics, in particular the development of micromaser techniques[9-14]. In these experiments one can ensure that an atom passing through a cavity will make a transition from an excited state to a lower state because of the interaction with the photons in the cavity.

To appreciate the logic of the present scheme, let us consider a beam of atoms replacing the light beam in the Einstein–Bohr dialogue, and the electron beam in the Feynman example. Just as in the previous cases, a beam of atoms incident upon a two-slit arrangement will show an interference pattern. As indicated in Fig. 3, a series of wider slits is used as collimators to define two atomic beams that arrive at the narrow slits where the interference pattern originates.

Let us disregard for the moment the laser and the maser cavities indicated in Fig. 3. In the interference region, the wave function describing the centre-of-mass motion of the atoms is then the sum of two terms referring to the two slits

$$\Psi(\mathbf{r}) = \frac{1}{\sqrt{2}}[\psi_1(\mathbf{r}) + \psi_2(\mathbf{r})] \quad (2)$$

and the probability density of particles falling on the screen where $\mathbf{r}=\mathbf{R}$, denoted by $P(\mathbf{R})$, will be given by the squared modulus of $\Psi(\mathbf{R})$, that is

$$P(\mathbf{R}) = \tfrac{1}{2}[|\psi_1(\mathbf{R})|^2 + |\psi_2(\mathbf{R})|^2 + \psi_1(\mathbf{R})^*\psi_2(\mathbf{R}) + \psi_2(\mathbf{R})^*\psi_1(\mathbf{R})] \quad (3)$$

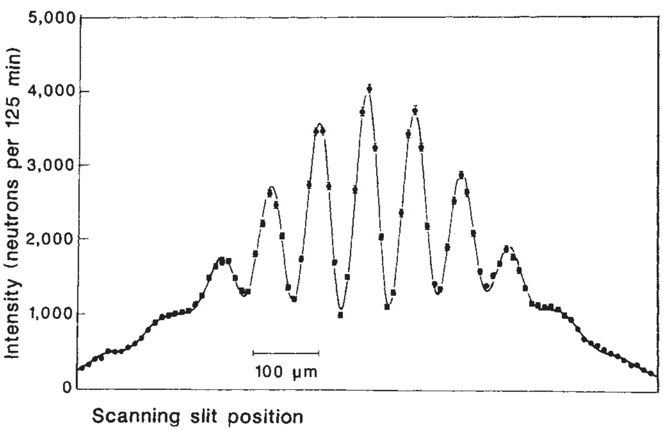

FIG. 2 Interference pattern produced by slow neutrons passing through a double slit. The solid curve represents the quantum mechanical prediction without any fitting. This plot is taken from ref. 34.

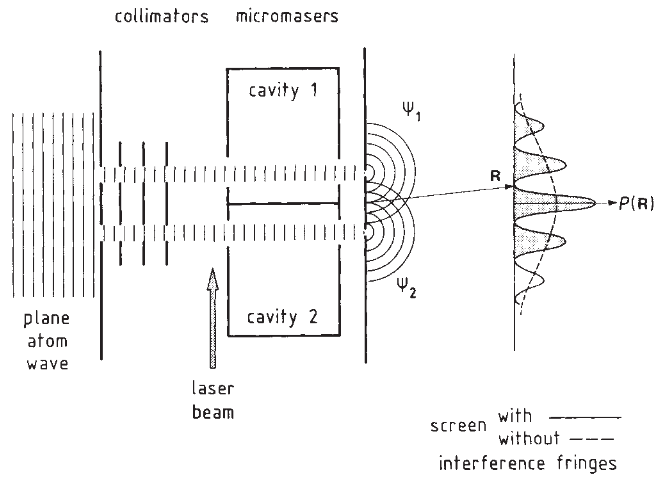

FIG. 3 Two-slit experiment with atoms. A set of wider slits collimates two atom beams which illuminate the narrow slits where the interference pattern originates. The collimation of the atomic beams would actually be done using atomic optics. One could, for instance, employ six-pole fields operating either on the magnetic dipole moment, or in the case of Rydberg atoms on the field-induced electric dipole moment. This set-up is supplemented by two high-quality micromaser cavities and a laser beam to to provide which-path information.

We note that the usual interference behaviour is represented by the cross-terms $\psi_1^*\psi_2+\psi_2^*\psi_1$.

Now just as Feynman's electron beam provided us with another handle on the interfering particles, not present in the case of interfering light beams, so, in the present case of an atomic system, we have further degrees of freedom involving the internal structure of the atom that are not available to us in the electron beam example.

In fact, we can now envisage preparing the atomic beam in an excited state (with the aid of a suitably operated laser) and then allowing the atoms to pass through the maser cavities in Fig. 3. On traversing either one of the cavities, the atom will emit a microwave photon and could leave *welcher Weg* information in the cavity.

One might think that the process of interacting with the microwave cavity fields and spontaneously emitting photons would disturb the centre-of-mass wave functions $\psi_1(\mathbf{r})$ and $\psi_2(\mathbf{r})$. The careful calculation reported in ref. 20 shows that this is not true. The natural way to discuss the centre-of-mass motion (the only essential parameter to be considered here) is in terms of kinetic and potential energy. In this language, the coupling between the atom and either one of the quantized cavity fields appears as a very small potential energy, whose sign and magnitude depends on the internal atomic and photonic quantum numbers. The wave function then consists of two components, one exposed to a weak attractive potential and the other to a repulsive one; the dynamical difference between attraction and repulsion effects the internal atomic transition accompanied by the emission of a photon. After the atom has traversed the cavity, it is again in force-free space and its momentum has the initial value. Thus, no net momentum is transferred to the atom during the interaction with the cavity fields. The de Broglie wave length of the atom is, therefore, not affected when a cavity photon is emitted, and so we have here an experiment which is "so delicate that it does not disturb" the interference pattern. (We should mention here that it is not possible to associate a definite momentum with a cavity mode, as this is not defined for a localized photon. Therefore the discussion of the atom–field interaction cannot be carried out on the basis of momentum transfer.)

In this sense we have conceived a *welcher Weg* detector which does not fall prey to the position–momentum uncertainty relation. How then are we to deal with the issue of complementarity? As discussed in the next section, it is simply the correlations between the detectors (micromaser cavities) and the atomic beams which are responsible for the loss of coherence (interference fringes) in the present experimental arrangement.

The above discussion of the *welcher Weg* detector was based on an atomic interference experiment. We should mention here that other experimental arrangements are also possible. For example, the two-field method developed by Norman Ramsey[21] can be applied. If the two fields necessary for this method are produced in identical micromaser cavities which are being traversed by the atoms one after the other, then equation (3) applies as well. The quantum-beat experiment proposed in ref. 19 is another possible scheme.

In the next section the micromaser *welcher Weg* detector is studied in more detail. In the section after next, we then ask what happens if one erases the which-path information contained in the *welcher Weg* micromaser cavities. Will interference fringes reappear?

A micromaser *welcher Weg* detector

A key ingredient in the micromaser *welcher Weg* detector is an excited atom which emits a photon when travelling through the cavity but not outside. (For a readable account of cavity electrodynamics, try ref. 22.) An atom in a long-lived Rydberg state, such as the $63p_{3/2}$ state of rubidium, is well suited to the present problem. In passing through the cavity it couples strongly either to $61d_{3/2}$ or to $61d_{5/2}$ at ~21 GHz as indicated in Fig. 4*a*. These states are currently used in micromaser experiments.

When such an atom is placed in a resonant cavity, it couples much more strongly to the microwave field and in fact decays rapidly from $63p_{3/2}$ (state a) to $61d_{5/2}$ (state b), for instance, because the mode density in the cavity (see Fig. 4*b*) is much larger than that in free space.

It is possible in principle, and realized in practice, for a Rydberg atom to make the transition $a\to b$ with unit probability when passing through the cavity, through spontaneous emission of a cavity microwave photon, even when the cavity does not contain photons initially.

Proceeding with the discussion of the micromaser *welcher Weg* detector, we return to Fig. 3 where, immediately preceding the masers, a laser beam is introduced, designed to excite all the atoms from the ground state to the excited state a. This can be accomplished by controlling the intensity of the laser beam such that this transition happens with certainty.

In the absence of the laser–cavities system, we now describe the atomic beam, after passing through the double slits, by the state vector

$$\Psi(\mathbf{r})=\frac{1}{\sqrt{2}}[\psi_1(\mathbf{r})+\psi_2(\mathbf{r})]|i\rangle \tag{4}$$

where $\mathbf{r}$ is the centre-of-mass coordinate and i denotes the internal state of the atom. Hence the probability density for particles on the screen at $\mathbf{r}=\mathbf{R}$ is given by the squared modulus of $\Psi(\mathbf{R})$,

$$P(\mathbf{R})=\tfrac{1}{2}[|\psi_1|^2+|\psi_2|^2+(\psi_1^*\psi_2+\psi_2^*\psi_1)]\langle i|i\rangle \tag{5}$$

which, of course, agrees with equation (3).

Next consider the situation with the laser turned on and the ultracold (vacuum) micromaser cavities put into the two paths, as in Fig. 3. Before entering the cavities, the laser beam excites the atoms to the long-lived Rydberg state a. After passing through the cavities and making the transition $a\to b$, say, by spontaneous emission of a photon, the state of the correlated atomic beam and maser cavity system is given by

$$\Psi(\mathbf{r})=\frac{1}{\sqrt{2}}[\psi_1(\mathbf{r})|1_1 0_2\rangle+\psi_2(\mathbf{r})|0_1 1_2\rangle]|b\rangle \tag{6}$$

where, for example, $1_1 0_2$ denotes the state in which there is one photon in cavity 1 and none in cavity 2. Please note that unlike (4) this $\Psi(\mathbf{r})$ is not a product of two factors, one referring to the atomic and the other to the photonic degrees of freedom. The system and the detector have become entangled by their

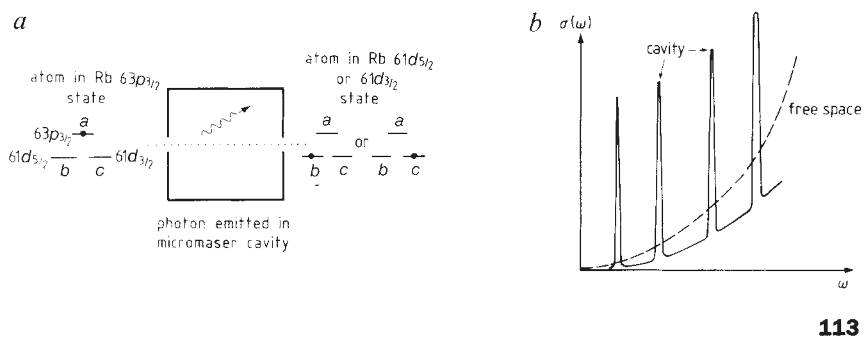

FIG. 4 *a*, Rubidium Rydberg atom in the $63p_{3/2}$ state *a* passes through a micromaser cavity, spontaneously emitting a microwave photon and making a transition to the $61d_{5/2}$ state *b* or the $61d_{3/2}$ state *c*. *b*, Density of photon modes, $\sigma(\omega)$, in free space (dashed curve) and in a micromaser cavity (solid curve), sketched as a function of the frequency ω.

interaction. In contrast to equation (3), the probability density at the screen is now given by

$$P(\mathbf{R}) = \tfrac{1}{2}[|\psi_1|^2 + |\psi_2|^2 + \psi_1^* \psi_2 \langle 1_1 0_2 | 0_1 1_2 \rangle + \psi_2^* \psi_1 \langle 0_1 1_2 | 1_1 0_2 \rangle] \langle b | b \rangle \quad (7)$$

But because $\langle 1_1 0_2 | 0_1 1_2 \rangle$ vanishes, the interference terms disappear here, so that

$$P(\mathbf{R}) = \tfrac{1}{2}[|\psi_1|^2 + |\psi_2|^2] \quad (8)$$

does not show fringes.

The micromasers will serve as *welcher Weg* detectors only if the one extra photon left by the atom changes the photon field in a detectable manner. Thus whether which-path information is available or not depends on the photon states initially prepared in the cavities. One extreme situation has just been discussed: no photons initially, one photon in one of the detectors finally. Clearly, here one can tell through which cavity, and therefore through which slit, the atom came to the screen. The situation is quite different when the cavities contain classical microwave radiation with large (average) numbers of photons, N_1 and N_2, which have spreads given by their square roots. For instance, the change in photon number in cavity 1 is now from $N_1 \pm \sqrt{N_1}$ to $N_1 + 1 \pm \sqrt{N_1}$. This change cannot be detected, because $\sqrt{N_1} \gg 1$, so that there is no which-path information available. (For more details about *welcher Weg* detectors, consult ref. 23.)

In the latter situation (classical radiation in the micromaser cavities), we cannot tell through which slit the atom reached the screen and the interference pattern is just the same as in the absence of the micromaser cavities. In contrast, cavities containing no photons initially store which-path information and therefore the interference pattern is lost. It is changed to the incoherent superposition (8) of one-slit patterns.

We emphasize once more that the micromaser *welcher Weg* detectors are recoil-free; there is no significant change in the spatial wave function of the atoms. It is the correlation of the centre-of-mass wave function to the photon degrees of freedom in the cavities that is responsible for the loss of interference.

In this context, we point out related neutron[24,25] experiments in which radio-frequency fields are employed to change the direction of the magnetic moment and thus the spin state: this does not affect the interference properties of the neutrons. These experiments therefore demonstrate that we can indeed manipulate internal degrees of freedom without changing the centre-of-mass wave function of a quantum system.

BOX 1 The Einstein-Bohr recoiling slit problem

Photons arriving on the (distant) screen of Fig. 1*a* at the location of the first side maximum of the fringe pattern, a distance Δx from the central maximum, possess different momenta $\hbar\mathbf{k}_1$ or $\hbar\mathbf{k}_2$ depending through which slit they reach the screen. The difference Δk_x of the x-component is well approximated by

$$\Delta k_x \approx \frac{2\pi}{\Delta x} \quad \text{(a)}$$

The recoil momentum of the plate supporting the slits must therefore be determined with a precision $\Delta p_x \sim \hbar \Delta k_x$, at least, to be able to tell through which slit the photon reached the screen. Thus $\hbar \Delta k_x$ must be distinctly larger than δp_x, the uncertainty in momentum of the slit plate, so that Heisenberg's uncertainty relation

$$\delta p_x \sim \frac{\hbar}{\delta x} \quad \text{(b)}$$

where δx is the uncertainty in position of the slit plate, implies

$$\Delta k_x > \frac{1}{\delta x} \quad \text{(c)}$$

In view of (a) this tells us that

$$\delta x \geqslant \Delta x \quad \text{(d)}$$

stating that the uncertainty in locating the slits (and therefore the fringes) is larger than the spacing between the fringes. In other words, the fringe pattern is washed out.

Quantum eraser

In the preceding section we have seen that it is the system-detector correlations which account for the dramatic effects of the measuring apparatus on the system of interest. It is no surprise that coherence is destroyed as soon as one has *welcher Weg* information, but here no uncontrollable scattering events (as in Fig. 1*a*; see box) were involved in destroying the interference (wave-like) behaviour.

One then wonders whether it might not be possible to retrieve the coherent interference cross-terms by removing ('erasing') the *welcher Weg* information contained in the detectors. In this sense, we are here considering the quantum eraser problem as discussed sometime ago by M.O.S.[26] (inspired by John Wheeler's suggestion[27] of delayed-choice experiments) and also by others[28-31]. If we erase the *welcher Weg* information in the microwave cavities, will spin coherence be restored? Notice that if we considered the coherence to be lost because of a random scattering or other stochastic perturbations, as studied in refs 32 and 33, for example, this question would never come up.

In fact, we shall see that interference effects can be restored by manipulating the *welcher Weg* detectors long after the atoms have passed. Edwin Jaynes[29] made some memorable remarks on this problem, which if adapted to the present context would read:

> We have, then, the full EPR [Einstein–Podolsky–Rosen] paradox—and more. By applying or not applying the eraser mechanism before measuring the state of the microwave cavities we can, at will, force the atomic beam into either: (1) a state with a known path, and no possibility of interference effects in any subsequent measurement; (2) a state with both ψ_1 and ψ_2 present with a known relative phase. Interference effects are then not only observable, but predictable. And we can decide which to do after the interaction is over and the atom is far from the cavities, so there can be no thought of any physical influence on the atom's centre-of-mass wavefunction!
>
> From this, it is pretty clear that present quantum theory not only does not use—it does not even dare to mention—the notion of a 'real physical situation'. Defenders of the theory say that this notion is philosophically naive, a throwback to outmoded ways of thinking, and that recognition of this constitutes deep new wisdom about the nature of human knowledge. I say that it constitutes a violent irrationality, that somewhere in this theory the distinction between reality and our knowledge of reality has become lost, and the result has more the character of medieval necromancy than of science. It has been my hope that quantum optics, with its vast new technological capability, might be able to provide the experimental clue that will show us to resolve these contradictions.

In the following we take up Jaynes' challenge, showing how to resolve this "paradox" and suggest further tests of complementarity in quantum mechanics within the framework of modern quantum optics. We present a gedanken experiment involving shutters and ideal photodetectors having unit quantum efficiency. This simple scheme is easy to understand and makes the physics clear, although it may not be possible to realize experimentally. Alternative schemes based on further application of the atomic beam(s)/micromaser combination, which we hope are more experimentally feasible, will be published elsewhere.

Consider now the arrangement of the atomic beam/micromaser system as indicated in Fig. 5*a*. There we see that the atoms pass through the two maser cavity detectors, but now we will imagine that the *welcher Weg* detectors are separated by a shutter-detector combination. So we now have a configuration in which the quantum eraser becomes possible. In particular, consider the cavity system in Fig. 5*a*. There we see two shutters arranged such that radiation will be constrained to remain either in the upper or the lower cavity, when the shutters are closed.

We further imagine that on opening the shutters, light will be allowed to interact with the photodetector wall. In this way the radiation, which is left either in the upper or in the lower cavity, depending upon whether the atom travelled along the upper or lower path, will now be absorbed and the 'memory of passage' (the *welcher Weg* information) could be said to be erased.

Do we now (after erasure) regain interference fringes? The answer is yes, but how can that be? The atoms are now far removed from the micromaser cavities and so "there can be no thought of any physical influence on the atom's centre-of-mass wave function". The answer to this question is given mathematically as follows.

Extending the mathematical description to include the detector, which is initially in its ground state d, we have

$$\Psi(\mathbf{r}) = \frac{1}{\sqrt{2}}[\psi_1(\mathbf{r})|1_1 0_2\rangle + \psi_2(\mathbf{r})|0_1 1_2\rangle]|b\rangle|d\rangle \tag{9}$$

which replaces equation (6). After absorbing a photon, the detector would be found in the excited state e.

It is now convenient to introduce symmetric, ψ_+, and antisymmetric, ψ_-, atomic states defined as

$$\psi_\pm(\mathbf{r}) = \frac{1}{\sqrt{2}}[\psi_1(\mathbf{r}) \pm \psi_2(\mathbf{r})] \tag{10}$$

Likewise, we introduce symmetric, $|+\rangle$, and antisymmetric, $|-\rangle$, states of the radiation fields contained in the *welcher Weg* cavities,

$$|\pm\rangle = \frac{1}{\sqrt{2}}[|1_1 0_2\rangle \pm |0_1 1_2\rangle] \tag{11}$$

In terms of equations (10) and (11), the state (9) of the atom-beam/microwave-cavity/detector system appears as

$$\Psi(\mathbf{r}) = \frac{1}{\sqrt{2}}[\psi_+(\mathbf{r})|+\rangle + \psi_-(\mathbf{r})|-\rangle]|b\rangle|d\rangle \tag{12}$$

We now consider the interaction between the radiation field existing in the cavity and the detector. As mentioned earlier, we envisage the detector to consist of an atom with a lower state d and an excited state e. The interaction hamiltonian between field and detector depends on symmetric combinations of the field variables, so that only the symmetric state $|+\rangle$ will couple to the fields.

We then find that the action of the detector (eraser) system produces the state

$$\Psi(\mathbf{r}) = \frac{1}{\sqrt{2}}[\psi_+(\mathbf{r})|0_1 0_2\rangle|e\rangle + \psi_-(\mathbf{r})|-\rangle|d\rangle]|b\rangle \tag{13}$$

That is, the symmetric interaction couples only to the symmetric radiation state $|+\rangle$; the antisymmetric state $|-\rangle$ remains unchanged.

Now, the atomic probability density at the screen goes as

$$P(\mathbf{R}) = \tfrac{1}{2}[\psi_+^*(\mathbf{R})\psi_+(\mathbf{R}) + \psi_-^*(\mathbf{R})\psi_-(\mathbf{R})]$$
$$= \tfrac{1}{2}[\psi_1^*(\mathbf{R})\psi_1(\mathbf{R}) + \psi_2^*(\mathbf{R})\psi_2(\mathbf{R})] \tag{14}$$

and does not show any interference fringes as long as the final state of the detector is unknown. But if one asks what is the probability density $P_e(\mathbf{R})$ for finding both the detector excited and the atom at $\mathbf{R}$ on the screen, the answer is

$$P_e(\mathbf{R}) = |\psi_+(\mathbf{R})|^2$$
$$= \tfrac{1}{2}[|\psi_1(\mathbf{R})|^2 + |\psi_2(\mathbf{R})|^2] + \mathrm{Re}\,[\psi_1^*(\mathbf{R})\psi_2(\mathbf{R})] \tag{15}$$

which exhibits the same fringes as equation (3), indicated as a solid line in Fig. 5*b*. In contrast, the probability density $P_d(\mathbf{R})$ for finding both the detector deexcited and the atom at $\mathbf{R}$ on the screen is

$$P_d(\mathbf{R}) = |\psi_-(\mathbf{R})|^2$$
$$= \tfrac{1}{2}[|\psi_1(\mathbf{R})|^2 + |\psi_2(\mathbf{R})|^2] - \mathrm{Re}\,[\psi_1^*(\mathbf{R})\psi_2(\mathbf{R})] \tag{16}$$

giving rise to the antifringes indicated by the broken line in Fig. 5*b*. If the eraser photon signal is disregarded, one obtains the superposition (14), equal to half the sum of P_e and P_d, which is fringeless, and, of course, identical with (8).

Here is the physical interpretation of this calculation. After an atom has run the gauntlet from the oven to the screen, passing through micromasers and leaving its tell-tale photon, we record an event somewhere on the screen. Then we return to the *welcher Weg* micromasers, open the shutters and allow the absorption of the microwave photon. When we observe a photocount in the detector we know that erasure has been completed. In this event the atom is counted as a 'yes'-atom.

Then we wait for another atom to pass through the system from oven to screen. Again we record an event on the screen and then turn to the micromaser cavities. This time suppose that, upon opening the shutter, we observe no photocount in the quantum eraser detector. This will be the case half of the time, as explained above. Now we count the atom as a 'no'-atom.

We repeat the above sequence many times. Eventually, the 'yes'-atoms will build up the solid-line fringes in Fig. 5*b*, and the 'no'-atoms produce the broken-line antifringes. Finally we note that the fringes and antifringes will cancel if we do not correlate them to the state of the eraser-detector. In this way we have resolved the 'Jaynes paradox'.

Having presented the physics of quantum erasure we now turn to an experimentally more realizable scheme which has much in common with the quantum eraser idea. We consider, as in Fig. 6, the asymmetric situation in which cavity 1 is tuned to the transition $a \rightarrow b$ ($63p_{3/2} \rightarrow 61d_{3/2}$), and cavity 2 is tuned to the transition $a \rightarrow c$ ($63p_{3/2} \rightarrow 61d_{5/2}$).

Even if the cavities contain classical microwave radiation, as we shall assume in the sequel, and therefore do not store which-path information, the screen will not show interference fringes because the internal atomic states b and c are orthogonal. This is analogous to the disappearance of the interference terms in equation (7), except that now the atoms themselves carry the *welcher Weg* information.

The latter circumstance again invites the question: could one not induce the transitions $b \rightarrow c$ in the atoms that traversed

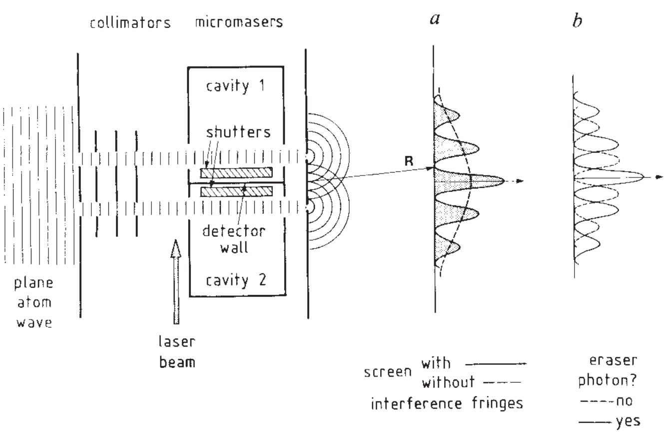

FIG. 5 *a*, Quantum erasure configuration in which electro-optic shutters separate microwave photons in two cavities from the thin-film semiconductor (detector wall) which absorbs microwave photons and acts as a photodetector. *b*, Density of particles on the screen depending upon whether a photocount is observed in the detector wall ('yes') or not ('no'), demonstrating that correlations between the event on the screen and the eraser photocount are necessary to retrieve the interference pattern.

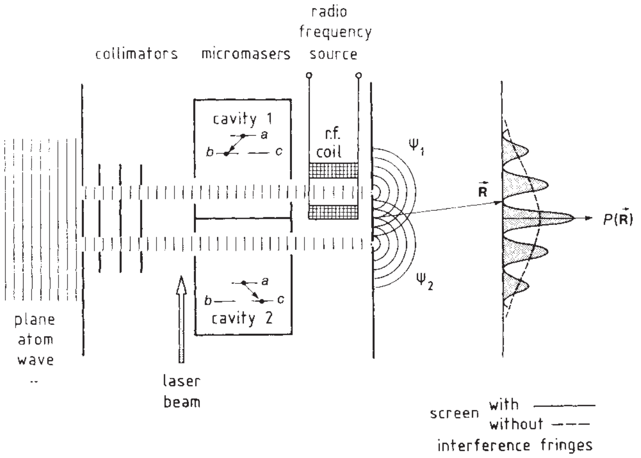

FIG. 6 Asymmetric set-up in which cavity 1 induces the transition $a \to b$ and cavity 2 induces $a \to c$. Which-path information is erased by the radio frequency in the coil where $b \to c$ happens.

cavity 1, so that the which-path information is erased, and thereby make the interference pattern reappear? The answer is affirmative. The actual experimental realization, however, is a delicate matter, because one must exert careful control on the phases of the various classical radiation fields. To appreciate what is involved, suppose that between cavity 1 and the slit plate there is a coil that can be fed with radio frequency of $\sim$50 MHz with the right strength to ensure the transition $b \to c$, as depicted in Fig. 6. In the interference region the state of the atom is essentially

$$\Psi(\mathbf{r}) = \frac{1}{\sqrt{2}}[\psi_1(\mathbf{r}) + e^{i\beta}\psi_2(\mathbf{r})]|c\rangle \qquad (17)$$

where the relative phase angle β is determined by the phases of the microwave fields in the two maser cavities and the radio-wave field in the coil. As these fields have different frequencies, β really refers to a certain instant, the moment, say, when the atom is excited to state a by the laser beam. The probability density at the screen

$$P(\mathbf{R}) = |\Psi(\mathbf{R})|^2 = \tfrac{1}{2}(|\psi_1|^2 + |\psi_2|^2) + \mathrm{Re}\,(\psi_1^* e^{i\beta}\psi_2) \qquad (18)$$

now exhibits an interference term that depends on β very sensitively. If, therefore, the value of β varies from atom to atom, the interference pattern will not build up. This illustrates quite well the omnipresent phenomenon of coherence loss caused by random phases. Consequently, one must ensure that the phase angle β is the same for all atoms to make the interference fringes reappear. In the set-up of Fig. 6 this can be achieved by adjusting the phase of the radiofrequency radiation in the coil to the phases that the microwave fields in the cavities have at the moment when the laser excites the atom. An additional bonus is the possibility of varying the chosen value of β, which enables one to shift the interference pattern on the screen. In summary, the control over the phase angle β represents a switch with which the experimenter can turn the interference fringes on and off, or relocate them.

Thus it would seem that the way is open for experiments of the *welcher Weg*/quantum-eraser type. No doubt they will be difficult, but as we gain more experience with these 'amazing masers', experiments along these lines will one day be realized, and *welcher-Weg*-type experiments are now under way at the Max-Planck-Institut in Garching. □

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