Put two non-interacting particles of mass M in a one-dimensional harmonic oscillator potential of frequency ω. The two particle wave function is:

$$\Psi(x_1, x_2) = \psi_n(x_1) * \psi_m(x_2)$$

Where ψ_n and ψ_m are the n'th and m'th Harmonic oscillator eigenfunction and x_1 and x_2 are positions for the two particles. We can write the wave functions as the following.

$$\psi_n(x_1) = \frac{1}{\sqrt{2^n * n!}} * \left(\frac{M\omega}{\pi\hbar}\right)^{\frac{1}{4}} * e^{-\frac{M\omega x_1^2}{2\hbar}} * H_n(y)$$

$$\psi_m(x_2) = \frac{1}{\sqrt{2^m * m!}} * \left(\frac{M\omega}{\pi\hbar}\right)^{\frac{1}{4}} * e^{-\frac{M\omega x_2^2}{2\hbar}} * H_m(y)$$

Where $y = \sqrt{\frac{M\omega}{\pi}} * x$ for $x = x_1, x_2$

We need to first find the expectation value $\langle x_1 - x_2 \rangle$:

$$\langle x_1 - x_2 \rangle = \langle x_1 \rangle - \langle x_2 \rangle$$

$\langle x_1 \rangle$ and $\langle x_2 \rangle$ can be represented as:

$$\langle x_1 \rangle = \int_{-\infty}^{\infty} (\psi_n^*(x_1) * x_1 * \psi_n(x_1)) dx_1$$

$$\langle x_2 \rangle = \int_{-\infty}^{\infty} (\psi_m^*(x_2) * x_2 * \psi_m(x_2)) dx_2$$

$$\langle x_1 - x_2 \rangle = \int_{-\infty}^{\infty} (\psi_n^*(x_1) * x_1 * \psi_n(x_1)) dx_1 - \int_{-\infty}^{\infty} (\psi_m^*(x_2) * x_2 * \psi_m(x_2)) dx_2$$

Putting in the value we find out that these integrals become 0, so:

$$\langle x_1 - x_2 \rangle = 0$$

Since the expectation value is finding the probability of the particle around a point, and in this case, the probability of finding a particle to the right of the other particle is equal to the probability of finding it to the left, so it becomes zero at the exact point.

Now we will need to find $\langle (x_1 - x_2)^2 \rangle$:

$$\langle (x_1 - x_2)^2 \rangle = \langle x_1^2 - 2x_1x_2 + x_2^2 \rangle = \langle x_1^2 \rangle - \langle 2x_1x_2 \rangle + \langle x_2^2 \rangle = \langle x_1^2 \rangle - \langle 2x_1 \rangle \langle x_2 \rangle + \langle x_2^2 \rangle$$

So we can do this in steps. For not crowding this document we can take:

$$\left(\psi_n^*(x_1) * x_1 * \psi_n(x_1)\right) = N$$

$$\left(\psi_m^*(x_2) * x_2 * \psi_m(x_2)\right) = P$$

First let's find $\langle x_1^2 \rangle$:

$$\langle x_1^2 \rangle = \int_{-\infty}^{\infty} \left(\psi_n^*(x_1) * x_1^2 * \psi_n(x_1)\right) dx_1 = \int_{-\infty}^{\infty} N^2 dx_1$$

$$\langle x_1^2 \rangle = \frac{2^{-1-n} * \hbar * H_n(y)^2}{M\omega n!}$$

Now for $\langle 2x_1 \rangle \langle x_2 \rangle$:

$$\langle 2x_1 \rangle \langle x_2 \rangle = \int_{-\infty}^{\infty} (2N) dx_1 * \int_{-\infty}^{\infty} (P) dx_2 = 0$$

And finally for $\langle x_2^2 \rangle$:

$$\langle x_2^2 \rangle = \int_{-\infty}^{\infty} (\psi_m^*(x_2) * x_2 * \psi_m(x_2)) dx_2 = \int_{-\infty}^{\infty} (P) dx_2$$

$$\langle x_2^2 \rangle = \frac{2^{-1-m} * \hbar * H_m(y)^2}{M\omega m!}$$

When we add all these together then we get:

$$\langle x_1^2 \rangle - \langle 2x_1 \rangle \langle x_2 \rangle + \langle x_2^2 \rangle = \frac{2^{-1-n} * \hbar * H_n(y)^2}{M\omega n!} - 0 + \frac{2^{-1-m} * \hbar * H_m(y)^2}{M\omega m!}$$

Or simply:

$$\langle (x_1 - x_2)^2 \rangle = \langle x_1^2 \rangle + \langle x_2^2 \rangle = \frac{2^{-1-n} * \hbar * H_n(y)^2}{M\omega n!} + \frac{2^{-1-m} * \hbar * H_m(y)^2}{M\omega m!}$$

$$\langle (x_1 - x_2)^2 \rangle = \langle x_1^2 \rangle + \langle x_2^2 \rangle = \frac{\hbar}{M\omega} \left(\frac{2^{-1-n} * H_n(y)^2}{n!} + \frac{2^{-1-m} * H_m(y)^2}{m!} \right)$$