Freitag, 27. Februar 2009
23:48

Lecture (Context)

$z = f(x,y)$ then $\boxed{\Delta z \approx f_x \, \Delta x + f_y \, \Delta y}$

Justify this formula: tangent-plane approximation

plane to $z = f(x,y)$?

Know: f_x, f_y are slopes of two tangent lines to the graph

If $\frac{\partial f}{\partial x}(x_0, y_0) = a \Rightarrow L_1 \begin{cases} z = z_0 + a(x - x_0) \\ y = y_0 \quad (\text{const.}) \end{cases}$

$\frac{\partial f}{\partial y}(x_0, y_0) = b \Rightarrow L_2 = \begin{cases} z = z_0 + b(y - y_0) \\ x = x_0 \end{cases}$

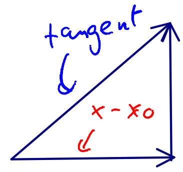
L_1, L_2 are both tangent to the graph $z = f(x,y)$

Together they determine a plane.

$$z = z_0 + a(x - x_0) + b(y - y_0)$$

Approximation formula says: graph of f is close to its tangent plane.

$Q_1 \, (x \,|\, y_0 \,|\, z0 + a(x - x_0))$



Q_0

$(x_0 \,|\, y_0 \,|\, z0)$

$\Rightarrow \overrightarrow{Q_0 Q_1} = \langle x - x_0, \ 0, \ a(x - x_0) \rangle$

$\Rightarrow \overrightarrow{Q_0 Q_2} = \langle 0, \ y - y_0, \ b(y - y_0) \rangle$

$\Rightarrow$ should be 2 vectors on the plane, but cross-product gives strange values