

If I want to differentiate a vector trilinear term $A^2 A_\alpha \equiv A^\nu A_\nu A_\alpha$

$$\partial_\mu (A^2 A_\alpha) \equiv \partial_\mu (A^\nu A_\nu A_\alpha)$$

I should first differentiate this expression first by A four-vector itself and then this A four-vector by coordinate: $\partial_\mu A$

But by which A it should be differentiated because differentiating by A_ν and by A_α gives different results respectively:

$$\text{differentiating by } A_\nu \quad (2A^\nu A_\alpha + A^2 \delta_\alpha^\nu) \partial_\mu A_\nu = 2A^\nu A_\alpha \partial_\mu A_\nu + A^2 \partial_\mu A_\alpha$$

$$\text{differentiating by } A_\alpha \quad \frac{\partial A^\nu A_\nu}{\partial A_\alpha} = 2A^\alpha \text{ and so } (2A^\alpha A_\alpha + A^2) \partial_\mu A_\alpha = 3A^2 \partial_\mu A_\alpha$$