

Question:

Consider the line integral defined by

$J = \int_C \mathbf{G} \cdot d\mathbf{r}$, with $\mathbf{G} = ye^{xy}\mathbf{\hat{a}}_x + (xe^{xy} + 1)\mathbf{\hat{a}}_y$, and the curve C extending from $x = 1$ to $x = 6$, along $y = x^3$

- (i) Evaluate the integral explicitly
- (ii) By taking the curl of \mathbf{G} , show that it is a conservative vector
- (iii) Integrate \mathbf{G} in order to determine the corresponding potential function Φ , such that
- (iv)

$$G_x = \partial\Phi/\partial x, \text{ and } G_y = \partial\Phi/\partial y$$

- (v) Verify the result of your integration by evaluating Φ at the end points of C .

Attempt so far:

Integrating $ye^{xy}\mathbf{\hat{a}}_x$ with respect to $x = ye^{xy}dx$

Integrating $(xe^{xy} + 1)\mathbf{\hat{a}}_y$ with respect to $y = (xe^{xy} + 1)dy$

$$dx = du$$

$$dy = 3u^2 du$$

Substituting

$$\text{Integral } ye^{u^4} du + 3u^3 e^{u^4} du + 3u^2 du$$

No further clue.