

Consider the continuous-time processing system in figure 1, which has two inputs and one output. The linear sub-system H is characterised by the impulse response $h(t) = e^{-2t}$, where t denotes time.

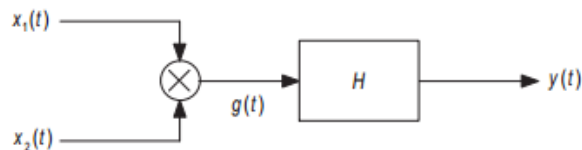


Fig. 1: Signal diagram. The signal $g(t)$ is the result of the multiplication of the two input signals $x_1(t)$ and $x_2(t)$; the output $y(t)$ is the result of passing $g(t)$ through the linear sub-system H .

- (a) Give a mathematical proof that this is a nonlinear system.
 (b) Consider the sinusoidal inputs $x_1(t) = \cos(\omega_1 t)$, $x_2(t) = \cos(\omega_2 t)$. Show that the signal $g(t)$ can be written as a sum of two cosine functions:

$$g(t) = \frac{1}{2} \cos[(\omega_2 - \omega_1)t] + \frac{1}{2} \cos[(\omega_2 + \omega_1)t]. \quad (1)$$

- (c) Give the differential equation for the linear sub-system H .
 (d) For the inputs defined under (b), derive a formula for the output signal y as a function of t .

(b) $\sin^2(x) = \frac{1}{2}[1 - \cos(2x)]$

$$\cos^2(x) = \frac{1}{2}[1 + \cos(2x)]$$

$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

$$\cos(a) + \cos(b) = 2\cos\frac{a+b}{2}\cos\frac{a-b}{2}$$

$$\cos(a)\cos(b) = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$$

Let $a = \omega_1 t$, $b = \omega_2 t$

Then $\cos(\omega_1 t) + \cos(\omega_2 t) = 2\cos\frac{\omega_1 t + \omega_2 t}{2}\cos\frac{\omega_1 t - \omega_2 t}{2}$

$$\begin{aligned} & 2\cos\frac{\omega_1 t + \omega_2 t}{2}\cos\frac{\omega_1 t - \omega_2 t}{2} \\ &= \frac{2}{2}\left[\cos\left(\frac{\omega_1 t + \omega_2 t}{2} - \frac{\omega_1 t - \omega_2 t}{2}\right) + \cos\left(\frac{\omega_1 t + \omega_2 t}{2} + \frac{\omega_1 t - \omega_2 t}{2}\right)\right] \end{aligned}$$

Terms inside bracket cancel to leave $\omega_2 t$ LHS bracket and $\omega_1 t$ in right bracket

$= [\cos(\omega_2 t) + \cos(\omega_1 t)]$ ---- back to where I started but noticed switch of ω side which is included in question

Using $2\cos\frac{\omega_1 t + \omega_2 t}{2}\cos\frac{\omega_1 t - \omega_2 t}{2}$ &

$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

$$2\left(\cos\left(\frac{\omega_1 t}{2}\right)\cos\left(\frac{\omega_2 t}{2}\right) - \sin\left(\frac{\omega_1 t}{2}\right)\sin\left(\frac{\omega_2 t}{2}\right)\right) \cdot \left[\cos\left(\frac{\omega_1 t}{2}\right)\cos\left(\frac{\omega_2 t}{2}\right) + \sin\left(\frac{\omega_1 t}{2}\right)\sin\left(\frac{\omega_2 t}{2}\right)\right]$$

$$2\left(\cos^2\left(\frac{\omega_1 t}{2}\right)\cos^2\left(\frac{\omega_2 t}{2}\right) - \sin^2\left(\frac{\omega_1 t}{2}\right)\sin^2\left(\frac{\omega_2 t}{2}\right)\right)$$

Then use

$$\sin^2(x) = \frac{1}{2}[1 - \cos(2x)]$$

$$\cos^2(x) = \frac{1}{2}[1 + \cos(2x)]$$

$$2\left([0.5(1 + \cos(\omega_1 t))][0.5(1 + \cos(\omega_2 t))]\right) - \left([0.5(1 - \cos(\omega_1 t))][0.5(1 - \cos(\omega_2 t))]\right)$$

This expands to

$$\frac{1}{2}[\cos(\omega_2 t) + \cos(\omega_1 t)]$$

Becomes

$$\frac{1}{2}\left[2\cos\frac{\omega_2 t + \omega_1 t}{2}\cos\frac{\omega_2 t - \omega_1 t}{2}\right]$$

IF

$$\frac{1}{4}[2\cos(\omega_2 t + \omega_1 t)\cos(\omega_2 t - \omega_1 t)]$$

For some reason I have an inkling that I can pull the /2 from the cosine functions which would get me the final answer I am looking for, although I am not in any way convinced this is allowed and could be a major faux pas.