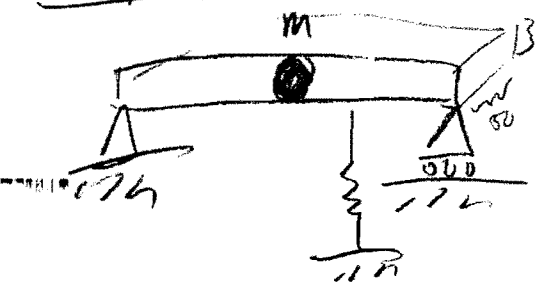
Example



$L = 0.75m$ $\quad \rho_a A = 2700 \times 0.05 \times 0.004$

$A = 50mm \times 4mm$ $\quad = 0.54\ kg/m$

$\rho_a = 2700\ kg/m^3$ $\quad \rho L = 0.41\ kg$

$$\phi_i(x) = A_i \sin \sigma_i x \qquad \sigma_i = \frac{n\pi}{l} = 4.189,\ 8.378,\ 12.567,\ \ldots$$

normalize

$$\int \rho \phi_i \phi_j \, dx = \int_0^l 0.54\, A_i^2 \sin^2 \sigma_i x \, dx = 1$$

$$= A_i^2 (0.54)\left(\frac{1}{2}\right) \int_0^{0.75} (1 - \cos 2\sigma_i x)\, dx$$

$$1 = A_i^2 \frac{0.54}{2} \left[(0.75 - 0) - \frac{1}{2\sigma_i} \left(\sin 2\sigma_i x \right) \Big|_0^{0.75} \right]$$

$$1 = A_i^2 \frac{0.54}{2} \left[(0.75) - \frac{1}{2\sigma_i} \sin(2\sigma_i \times 0.75) \right] \quad (\sin 0 = 0)$$

$$A_1 = \frac{1}{\sqrt{\frac{2.54}{2}(0.75)}} = 2.22$$

$$A_2 = 2.22$$

$$A_3 = 2.22$$

$$M = I + m \left[\phi_i \phi_j \right]_{x = 0.75/2 = 0.375} \qquad i, j = 1, \ldots n$$

$$\int \rho \phi_i \phi_j \, dx = \delta_{ij}$$

$$\phi_1(0.375) = 2.22\,(1.0) = 2.22$$

$$\phi_2(0.375) = 2.22 \sin(8.378 \times 0.375) = 0$$

$$\phi_3(0.375) = 2.22 \sin(12.567 \times 0.375) = -2.22$$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + m \begin{bmatrix} 4.928 & 0 & -4.928 \\ 0 & 0 & 0 \\ \text{Sym} & & 4.928 \end{bmatrix}$$

Find ω's for $m = 0.04, 0.4, 4$.

$K = 0$

$$K = \Lambda K^t \; k\left[\phi_i \phi_j\right]\Big|_{x=0.3625 = 0.75\times 0.75}$$

$$\Lambda_{ic} = \begin{bmatrix} \omega_1^2 & & 0 \\ & \omega_2^2 & \\ 0 & & \omega_3^2 \end{bmatrix} \qquad \omega_i^2 = \sigma_i^4 \frac{EI}{\rho}$$

$$\omega_1^2 = (4.189\times 0.75)^4 \frac{18.4}{0.54\times 0.75^4} = (\sigma_1 L)^4 \frac{EI}{\rho A L^4}$$

$E = 69\times 10^9 \; N/m^2$

$I = \frac{1}{12} 50\times 4^3 = 266.7 \; mm^4 \frac{10^{-12}}{1 \; mm^4}$

$EI = 184\times 10^{-4} \; Nm^2$

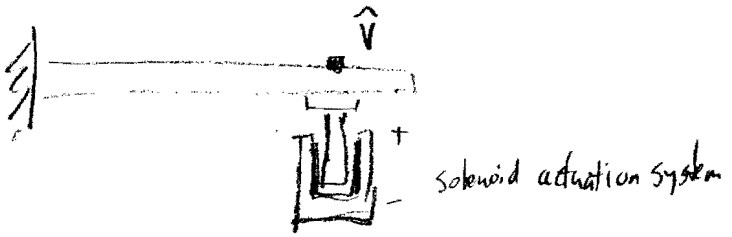
$= 18.4 \; Nm^2$ flexural rigidity

$\omega_1 = 102.43$ rad/sec $\quad f = 16.3$ Hz

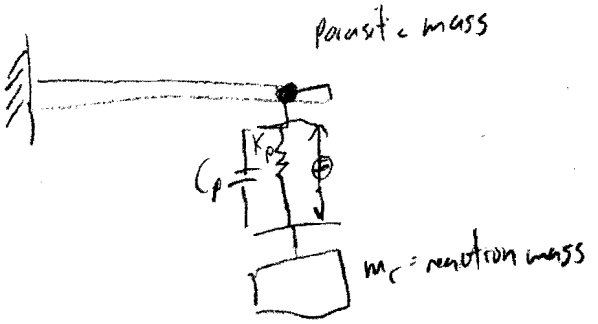
$\omega_2 = 409.7$ rad/sec $\quad f = 65.2$ Hz

$\omega_3 = 921.8$ rad/sec $\quad f = 146.7$ Hz

$$K = \begin{bmatrix} \omega_1^2 & 0 & 0 \\ 0 & \omega_2^2 & 0 \\ 0 & 0 & \omega_3^2 \end{bmatrix} + k \begin{bmatrix} 1.5696^2 & 1.5696\times 2.22 & 1.5696\times 1.5703 \\ & 2.22^2 & -2.22\times 1.5703 \\ \text{sym} & & 1.5703^2 \end{bmatrix}$$



solenoid actuation system



$$f(t) = -K_p\left(X_p - w(l_a, t)\right) - C_p\left(\dot{X}_p - \dot{w}(l_a, t)\right) + f_g(t)$$