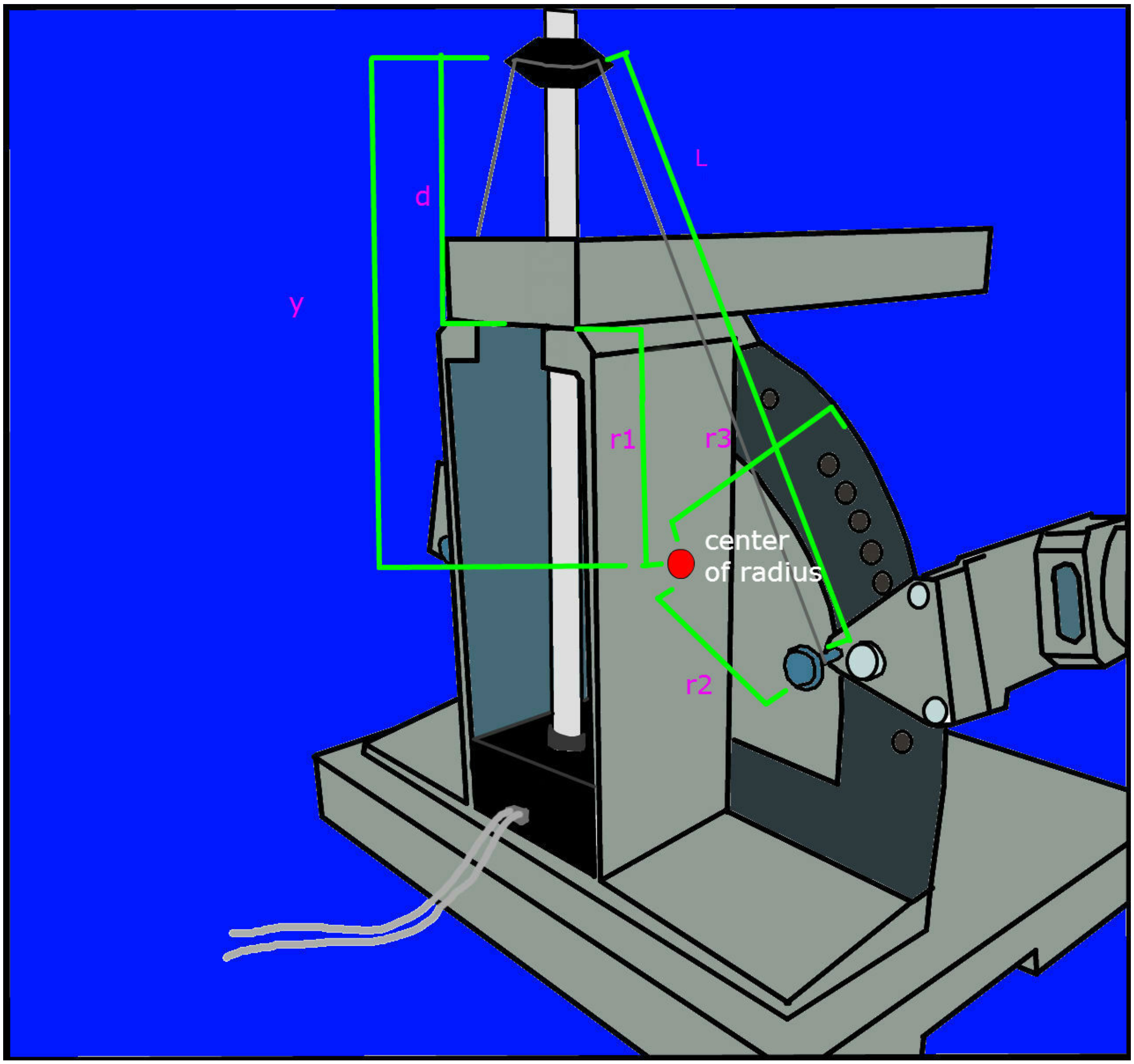
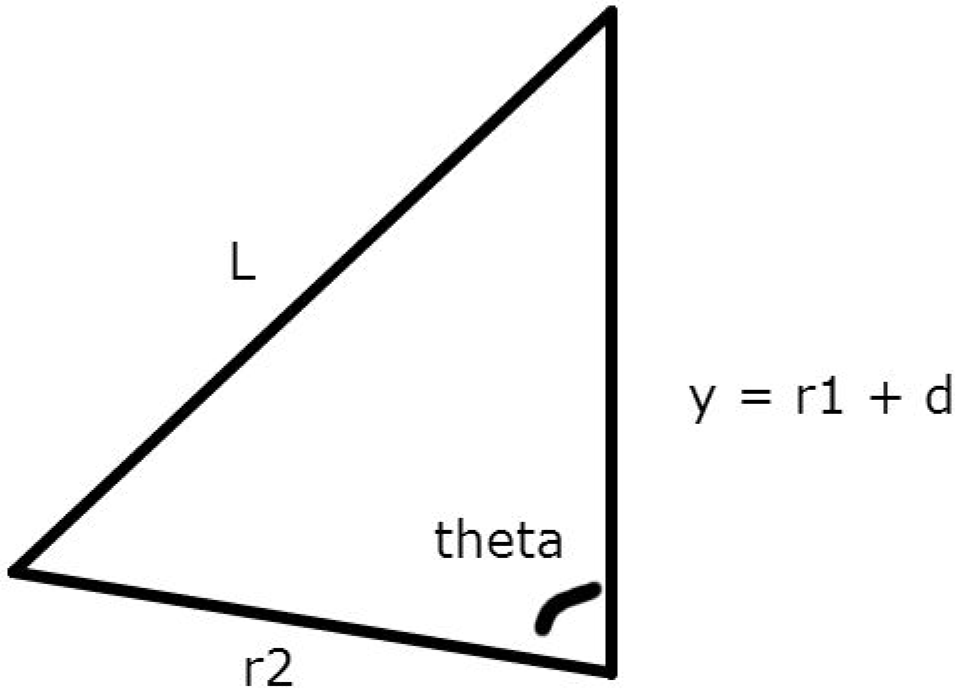
So I have (hopefully) derived an equation that describes the angle of incidence (θ) as a function of the distance above the main circle (d) and the length of the rope holding the laser arms (effectively the distance between the ropes two points of contact). I have drawn a diagram of the various variables, I will stay in variable form this entire derivation. The next time I come down I will actually measure these variables and perform some tests to ensure my derivation is accurate.



We are interested in the triangle formed by L, r2, and y.



Where θ is the angle opposite l. We can use the Law of Cosines to give us the equation:

$$L^2 = {r_2}^2 + y^2 - 2r_2 y cos\theta$$

Using this equation we can solve for θ, yielding

$$\theta = cos^{-1}\left(\frac{{r_2}^2+y^2-L^2}{2r_2 y}\right)$$

r2 is a fixed value, its just the distance from the center of rotation to the actual screw that is being pulled up by the lift mechanism. L is variable, but we will derive a value of L that satisfies θ from 30º to 90º, which is the total available range of angles. So in essence we can think of L as a constant. Y is the next variable to look at, since it is the total distance from the center of rotation to the connection point at the top of L, it is a variable. It depends on d, therefore we can rewrite the above equation for θ:

$$\theta = cos^{-1}\left(\frac{{r_2}^2+{r_1}^2+2r_1 d+d^2-L^2}{2r_2 r_1+2r_2 d}\right)$$

Now we have θ as just a function of one variable, namely d. This equation, will give us the angle at each value d, assuming the rest of the information is known about the setup. Next we wish to find the inverse equation so we can model the distance as a function of the desired angle. This turns out to be

$$d = r_2 cos\theta - r_1 \pm \sqrt{r_2{}^2 cos^2\theta + L^2 - r_2{}^2}$$

As θ moves from 90 to 30 degrees, we fully expect d to increase in size, so we want to use the plus sign instead of the minus sign.

$$d = r_2 cos\theta - r_1 + \sqrt{r_2{}^2 cos^2\theta + L^2 - r_2{}^2}$$

Now we would like to find a length L that will satisfy the range of θ from 30 to 90 degrees. This means we need to pick an L where d is greater than 0 throughout this range. We can do this by setting d = 0 and θ = 90º, then solving for L, this will give us the minimum length required for L to satisfy this range of θ.

$$0 \ = \ -r_1 + \sqrt{L^2 - r_2{}^2}$$
$$L \ = \ \sqrt{r_1{}^2 + r_2{}^2}$$

To keep from using a larger threaded rod, we should keep L to a minimum, so we will use this as the value of L. Now we can plug this back into the equation for d and the equation for θ

$$d = r_2 cos\theta - r_1 + \sqrt{r_2{}^2 cos^2\theta + r_1{}^2}$$
$$\theta = cos^{-1}\left(\frac{2r_1 d + d^2}{2r_2 r_1 + 2r_2 d}\right)$$

So to find the minimum value of d, namely when θ = 90º

$$d = 0$$

And for θ = 30º

$$d = \frac{\sqrt{3}}{2} r_2 - r_1 + \sqrt{\frac{3}{4} r_2{}^2 + r_1{}^2}$$

Now note that this value for d isn’t from the top of the machine, its from the bottom of the supporting plate that is the top part of the machine, so if this supporting plate has a thickness x, we could say that the top of the threaded rod needs to be a little taller than d - x from the above equation.

This is just a theoretical derivation of the required information. A lot of care will go into measuring precise values for r1 and r2.