

Disproof of the Riemann Hypothesis

By:

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Euler and others have shown that:

$$Z\left(\frac{1}{2} + it\right) = e^{\frac{i\gamma t + it \ln \pi}{2} - i \arctan(2t) + i \sum_{n=1}^{\infty} \left(\frac{t}{2n} - \arctan\left(\frac{2t}{4n+1}\right)\right)} 2^{\frac{1}{2} + it} \pi^{it - \frac{1}{2}} \sin\left(\frac{\pi\left(\frac{1}{2} + it\right)}{2}\right) \Gamma\left(1 - \left(\frac{1}{2} + it\right)\right) \zeta\left(1 - \left(\frac{1}{2} + it\right)\right)$$

$$1 = \frac{2 \Gamma\left(\frac{1}{2} + it\right)}{2\pi} \cos\left(\frac{\pi}{4} + \frac{\pi it}{2}\right)$$

If the Riemann Hypothesis were true. In order to prove the Riemann Hypothesis is true, we would need to prove the above statements.

Solving the first equation with respect to $\Gamma\left(\frac{1}{2} + it\right)$ gives:

$$\Gamma\left(\frac{1}{2} + it\right) = \frac{Z\left(\frac{1}{2} + it\right)}{e^{\frac{i\gamma t + it \ln \pi}{2} - i \arctan(2t) + i \sum_{n=1}^{\infty} \left(\frac{t}{2n} - \arctan\left(\frac{2t}{4n+1}\right)\right)} 2^{\frac{1}{2} + it} \pi^{it - \frac{1}{2}} \sin\left(\frac{\pi\left(\frac{1}{2} + it\right)}{2}\right) \Gamma\left(1 - \left(\frac{1}{2} + it\right)\right) \zeta\left(1 - \left(\frac{1}{2} + it\right)\right)}$$

Solving the second equation with respect to $\Gamma\left(\frac{1}{2} + it\right)$ gives:

$$\Gamma\left(\frac{1}{2} + it\right) = \frac{\pi}{\cos\left(\frac{\pi}{4} + \frac{\pi it}{2}\right)}$$

Now,

$$\Gamma\left(\frac{1}{2} + it\right) = \frac{\pi}{\cos\left(\frac{\pi}{4} + \frac{\pi it}{2}\right)} = \frac{Z\left(\frac{1}{2} + it\right)}{e^{\frac{i\gamma t + it \ln \pi}{2} - i \arctan(2t) + i \sum_{n=1}^{\infty} \left(\frac{t}{2n} - \arctan\left(\frac{2t}{4n+1}\right)\right)} 2^{\frac{1}{2} + it} \pi^{it - \frac{1}{2}} \sin\left(\frac{\pi\left(\frac{1}{2} + it\right)}{2}\right) \Gamma\left(1 - \left(\frac{1}{2} + it\right)\right) \zeta\left(1 - \left(\frac{1}{2} + it\right)\right)}$$

We can replace the Riemann Zeta function with the Z-function and cancel it out.

$$\frac{\cos\left(\frac{\pi}{4} + \frac{\pi it}{2}\right)}{\pi} = e^{\frac{i\gamma t + it \ln \pi}{2} - i \arctan(2t) + i \sum_{n=1}^{\infty} \left(\frac{t}{2n} - \arctan\left(\frac{2t}{4n+1}\right)\right)} 2^{\frac{1}{2} + it} \pi^{it - \frac{1}{2}} \sin\left(\frac{\pi}{4} + \frac{\pi it}{2}\right) \Gamma\left(\frac{1}{2} + it\right)$$

We could substitute Euler's constant with the Digamma function of 1.

$$\frac{\cos\left(\pi\left(\frac{4it+1}{4}\right)\right)}{\pi} = e^{\frac{\psi(1)it + it \ln \pi}{2} - i \arctan(2t) + i \sum_{n=1}^{\infty} \left(\frac{t}{2n} - \arctan\left(\frac{2t}{4n+1}\right)\right)} 2^{\frac{1}{2} + it} \pi^{it - \frac{1}{2}} \sin\left(\frac{\pi}{4} + \frac{\pi it}{2}\right) \Gamma\left(\frac{1}{2} + it\right)$$

$$\frac{\cos(\pi(\frac{4it+1}{4}))}{\pi} = e^{\frac{\frac{\Gamma'(1)}{\Gamma(1)}it+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))} 2^{\frac{1}{2}+it} \pi^{it-\frac{1}{2}} \sin(\frac{\pi}{4}+\frac{\pi it}{2})\Gamma(\frac{1}{2}+it)$$

$$\frac{\cos(\pi(\frac{4it+1}{4}))}{\pi} = e^{\frac{\frac{\Gamma'(1)}{\Gamma(1)}it+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))} 2^{\frac{1}{2}+it} \pi^{it-\frac{1}{2}} \sin(\pi(\frac{4it+1}{4}))\Gamma(\frac{1}{2}+it)$$

$$\pi = \frac{\cos(\pi(\frac{4it+1}{4}))}{e^{\frac{\frac{\Gamma'(1)}{\Gamma(1)}it+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))} 2^{\frac{1}{2}+it} \pi^{it-\frac{1}{2}} \sin(\pi(\frac{4it+1}{4}))\Gamma(\frac{1}{2}+it)}$$

$$\pi = \frac{\cos(\pi(\frac{4it+1}{4}))}{e^{\frac{\frac{d}{dt}(\ln(\Gamma(1))it+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))} 2^{\frac{1}{2}+it} \pi^{it-\frac{1}{2}} \sin(\pi(\frac{4it+1}{4}))\Gamma(\frac{1}{2}+it)}$$

$$\pi = \frac{\cos(\pi(\frac{4it+1}{4}))}{e^{\frac{\frac{d}{dt}(\ln(\int_0^{\infty} e^{\frac{1}{2}dt})it+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))} 2^{\frac{1}{2}+it} \pi^{it-\frac{1}{2}} \sin(\pi(\frac{4it+1}{4}))\int_0^{\infty} e^{-t^{\frac{1}{it-\frac{1}{2}}}} dt}$$

$$\pi = \frac{\cos(\pi(\frac{4it+1}{4}))}{e^{\frac{\frac{d}{dt}(\ln(\int_0^{\infty} ldt)it+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))} 2^{\frac{1}{2}+it} \pi^{it-\frac{1}{2}} \sin(\pi(\frac{4it+1}{4}))\int_0^{\infty} e^{-t^{\frac{1}{it-\frac{1}{2}}}} dt}$$

$$\pi = \frac{\cos(\pi(\frac{4it+1}{4}))}{e^{\frac{\frac{\ln(\frac{1}{2}+it)it+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))} 2^{\frac{1}{2}+it} \pi^{it-\frac{1}{2}} \sin(\pi(\frac{4it+1}{4}))\int_0^{\infty} e^{-t^{\frac{1}{it-\frac{1}{2}}}} dt}$$

$$\pi = \frac{\cos(\pi(\frac{4it+1}{4}))}{e^{\frac{\frac{\ln(\frac{1}{2}+it)it+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))} 2^{\frac{1}{2}+it} \pi^{it-\frac{1}{2}} \sin(\pi(\frac{4it+1}{4}))\frac{(2it-1)e^{-\frac{2t}{2it-1}}}{2}}$$

$$\pi = \frac{2\cos(\pi(\frac{4it+1}{4}))}{e^{\frac{\frac{\ln(\frac{1}{2}+it)-4t^4\ln\pi}{2(2it-1)}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))} 2^{\frac{1}{2}+it} \pi^{it-\frac{1}{2}} \sin(\pi(\frac{4it+1}{4}))(2it-1)}$$

$$\pi = \frac{2 \cot(\pi(\frac{4it+1}{4}))}{e^{-\left(\frac{\ln(\frac{1}{2}+it)-4t^4 \ln \pi}{2(2it-1)} - i \arctan(2t) + i \sum_{n=1}^{\infty} (\frac{t}{2n} - \arctan(\frac{2t}{4n+1}))\right)} 2^{\frac{1}{2}+it} \pi^{it-\frac{1}{2}} (2it-1)}$$

$$\pi = \frac{2 \cot(\pi(\frac{4it+1}{4}))}{e^{-\left(\frac{\ln(\frac{1}{2}+it)-4t^4 \ln \pi}{4it-2} - i \arctan(2t) + i \sum_{n=1}^{\infty} (\frac{t}{2n} - \arctan(\frac{2t}{4n+1}))\right)} 2^{\frac{1}{2}+it} \pi^{it-\frac{1}{2}} (2it-1)}$$

$$\pi = \frac{2 \cot(\pi(\frac{4it+1}{4}))}{e^{-\left(\frac{\ln(\frac{1}{2}+it)-4t^4 \ln \pi}{4it-2} - i \arctan(2t) + i \sum_{n=1}^{\infty} (\frac{t}{2n} - \arctan(\frac{2t}{4n+1}))\right)} (2^{\frac{3}{2}} it - \sqrt{2})(2^{it} \pi^{\frac{2it-1}{2}})}$$

$$\pi = \frac{2 \cot(\frac{4it\pi + \pi}{4})}{e^{-\left(\frac{\ln(\frac{1}{2}+it)-4t^4 \ln \pi}{4it-2} - i \arctan(2t) + i \sum_{n=1}^{\infty} (\frac{t}{2n} - \arctan(\frac{2t}{4n+1}))\right)} (2^{\frac{3}{2}} it - \sqrt{2})(2^{it} \pi^{\frac{2it-1}{2}})}$$

Since $\sum_{n=1}^{\infty} (\frac{t}{2n} - \arctan(\frac{2t}{4n+1})) = \infty$,

$$\pi = \frac{2 \cot(\frac{4it\pi + \pi}{4})}{e^{-\left(\frac{\ln(\frac{1}{2}+it)-4t^4 \ln \pi}{4it-2} - i \arctan(2t) + \infty i\right)} (2^{\frac{3}{2}} it - \sqrt{2})(2^{it} \pi^{\frac{2it-1}{2}})}$$

$$\pi = \frac{2 \cot(\frac{4it\pi + \pi}{4})}{e^{-\infty} (2^{\frac{3}{2}} it - \sqrt{2})(2^{it} \pi^{\frac{2it-1}{2}})}$$

$$\pi = \frac{2 \cot(\frac{4it\pi + \pi}{4})}{0(2^{\frac{3}{2}} it - \sqrt{2})(2^{it} \pi^{\frac{2it-1}{2}})}$$

$$\pi = \frac{2 \cot(\frac{4it\pi + \pi}{4})}{0}$$

$$\pi = \infty$$

which is totally false. Thus, the Riemann Hypothesis is invalid.