An extension to the Riemann Hypothesis

By:

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Euler and others have shown that:

$$Z(\frac{1}{2}+it)=e^{\frac{i\gamma t+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))}2^{\frac{1}{2}+it}\pi^{it-\frac{1}{2}}\sin(\frac{\pi(\frac{1}{2}+it)}{2})\Gamma(1-(\frac{1}{2}+it))\zeta(1-(\frac{1}{2}+it))$$

$$1=\frac{2\,\Gamma(\frac{1}{2}+it)}{2\pi}\cos(\frac{\pi}{4}+\frac{\pi it}{2})$$

If the Riemann Hypothesis were true. In order to prove the Riemann Hypothesis is true, we would need to prove the above statements.

Solving the first equation with respect to $\Gamma(\frac{1}{2}+it)$ gives:

$$\Gamma(\frac{1}{2}+it)=\frac{Z(\frac{1}{2}+it)}{e^{\frac{i\gamma t+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))}2^{\frac{1}{2}+it}\pi^{it-\frac{1}{2}}\sin(\frac{\pi(\frac{1}{2}+it)}{2})\Gamma(1-(\frac{1}{2}+it))\zeta(1-(\frac{1}{2}+it))}$$

Solving the second equation with respect to $\Gamma(\frac{1}{2}+it)$ gives:

$$\Gamma(\frac{1}{2}+it)=\frac{\pi}{\cos(\frac{\pi}{4}+\frac{\pi it}{2})}$$

Now,

$$\Gamma(\frac{1}{2}+it)=\frac{\pi}{\cos(\frac{\pi}{4}+\frac{\pi it}{2})}=\frac{Z(\frac{1}{2}+it)}{e^{\frac{i\gamma t+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))}2^{\frac{1}{2}+it}\pi^{it-\frac{1}{2}}\sin(\frac{\pi(\frac{1}{2}+it)}{2})\Gamma(1-(\frac{1}{2}+it))\zeta(1-(\frac{1}{2}+it))}$$

We can replace the Riemann Zeta function with the Z-function and cancel it out.

$$\frac{\cos(\frac{\pi}{4}+\frac{\pi it}{2})}{\pi}=e^{\frac{i\gamma t+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))}2^{\frac{1}{2}+it}\pi^{it-\frac{1}{2}}\sin(\frac{\pi}{4}+\frac{\pi it}{2})\Gamma(\frac{1}{2}+it)$$

We could substitute Euler’s constant with the Digamma function of 1.

$$\frac{\cos(\pi(\frac{4it+1}{4}))}{\pi}=e^{\frac{\psi(1)it+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))}2^{\frac{1}{2}+it}\pi^{it-\frac{1}{2}}\sin(\frac{\pi}{4}+\frac{\pi it}{2})\Gamma(\frac{1}{2}+it)$$

$$\frac{\cos(\pi(\frac{4it+1}{4}))}{\pi}=e^{\frac{\frac{\Gamma'(1)}{\Gamma(1)}it+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))}2^{\frac{1}{2}+it}\pi^{it-\frac{1}{2}}\sin(\frac{\pi}{4}+\frac{\pi it}{2})\Gamma(\frac{1}{2}+it)$$

$$\frac{\cos(\pi(\frac{4it+1}{4}))}{\pi}=e^{\frac{\frac{\Gamma'(1)}{\Gamma(1)}it+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))}2^{\frac{1}{2}+it}\pi^{it-\frac{1}{2}}\sin(\pi(\frac{4it+1}{4}))\Gamma(\frac{1}{2}+it)$$

$$\pi=\frac{\cos(\pi(\frac{4it+1}{4}))}{e^{\frac{\frac{\Gamma'(1)}{\Gamma(1)}it+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))}2^{\frac{1}{2}+it}\pi^{it-\frac{1}{2}}\sin(\pi(\frac{4it+1}{4}))\Gamma(\frac{1}{2}+it)}$$

$$\pi=\frac{\cos(\pi(\frac{4it+1}{4}))}{e^{\frac{\frac{d}{dt}(\ln(\Gamma(1))it+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))}2^{\frac{1}{2}+it}\pi^{it-\frac{1}{2}}\sin(\pi(\frac{4it+1}{4}))\Gamma(\frac{1}{2}+it)}$$

$$\pi=\frac{\cos(\pi(\frac{4it+1}{4}))}{e^{\frac{\frac{d}{dt}(\ln(\int_0^{\infty}e^{0^{\frac{1}{\frac{1}{2}}}}dt)it+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))}2^{\frac{1}{2}+it}\pi^{it-\frac{1}{2}}\sin(\pi(\frac{4it+1}{4}))\int_0^{\infty}e^{-t^{\frac{1}{it-\frac{1}{2}}}}dt}$$

$$\pi=\frac{\cos(\pi(\frac{4it+1}{4}))}{e^{\frac{\frac{d}{dt}(\ln(\int_0^{\infty}1dt)it+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))}2^{\frac{1}{2}+it}\pi^{it-\frac{1}{2}}\sin(\pi(\frac{4it+1}{4}))\int_0^{\infty}e^{-t^{\frac{1}{it-\frac{1}{2}}}}dt}$$

$$\pi=\frac{\cos(\pi(\frac{4it+1}{4}))}{e^{\frac{\ln(\frac{1}{2}+it)it+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))}2^{\frac{1}{2}+it}\pi^{it-\frac{1}{2}}\sin(\pi(\frac{4it+1}{4}))\int_0^{\infty}e^{-t^{\frac{1}{it-\frac{1}{2}}}}dt}$$

$$\pi=\frac{\cos(\pi(\frac{4it+1}{4}))}{e^{\frac{\ln(\frac{1}{2}+it)it+it\ln\pi}{2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))}2^{\frac{1}{2}+it}\pi^{it-\frac{1}{2}}\sin(\pi(\frac{4it+1}{4}))\frac{(2it-1)e^{-\frac{2t}{2it-1}}}{2}}$$

$$\pi=\frac{2\cos(\pi(\frac{4it+1}{4}))}{e^{-(\frac{\ln(\frac{1}{2}+it)-4t^4\ln\pi}{2(2it-1)}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1})))}2^{\frac{1}{2}+it}\pi^{it-\frac{1}{2}}\sin(\pi(\frac{4it+1}{4}))(2it-1)}$$

$$\pi = \frac{2\cot(\pi(\frac{4it+1}{4}))}{e^{-(\frac{\ln(\frac{1}{2}+it)-4t^4\ln\pi}{2(2it-1)}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1})))}2^{\frac{1}{2}+it}\pi^{it-\frac{1}{2}}(2it-1)}$$

$$\pi = \frac{2\cot(\pi(\frac{4it+1}{4}))}{e^{-(\frac{\ln(\frac{1}{2}+it)-4t^4\ln\pi}{4it-2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1})))}2^{\frac{1}{2}+it}\pi^{it-\frac{1}{2}}(2it-1)}$$

$$\pi = \frac{2\cot(\pi(\frac{4it+1}{4}))}{e^{-(\frac{\ln(\frac{1}{2}+it)-4t^4\ln\pi}{4it-2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1}))}(2^{\frac{3}{2}}it-\sqrt{2})(2^{it}\pi^{\frac{2it-1}{2}})}$$

$$\pi = \frac{2\cot(\frac{4it\pi+\pi}{4}))}{e^{-(\frac{\ln(\frac{1}{2}+it)-4t^4\ln\pi}{4it-2}-i\arctan(2t)+i\sum_{n=1}^{\infty}(\frac{t}{2n}-\arctan(\frac{2t}{4n+1})))}(2^{\frac{3}{2}}it-\sqrt{2})(2^{it}\pi^{\frac{2it-1}{2}})}$$

This is an extension to the Riemann Hypothesis and if it is true, the Riemann Hypothesis is valid.