1. The Rindler Metric

In cylindrical coordinates the metric is

$$ds^2 = -z^2dt^2 + dz^2 + dr^2 + r^2d\phi^2 \tag{1}$$

The coordinate ranges are $0 < t, z, r < \infty$, $0 \le \phi < 2\pi$. There is a stationary Killing vector field $\xi^\mu = \partial_t$ and an axial one $\psi^\mu = \partial_\phi$. The conserved quantities are given by $\xi^\mu\xi_\mu = z^2$ and $\psi^\mu\psi_\mu = r^2$.

1.1. The Kinematic Decomposition

For the stationary congruence $u^\mu = (1/z)\partial_t$ the results are,

$$\Theta = 0, \quad \dot{u}_\mu = -\frac{1}{z}\,\partial_z, \quad \omega^\mu = 0, \quad \sigma_{\mu\nu} = 0 \tag{2}$$

Where $\omega^a = (1/2)\epsilon^{abmi}u_b\,\omega_{mi}$ and gives the axis of rotation.

2. A Shear-Free Rotating Congruence

The vector field is (from Peter Donis)

$$u^\mu = \frac{1}{\sqrt{1-r^2\,\omega^2}\,z}\,\partial_t + \frac{w}{\sqrt{1-r^2\,\omega^2}}\,\partial_\phi \tag{3}$$

where $g_{\mu\nu}u^\mu u^\nu = -1$ and ω depends on coordinate z.

2.1. The Kinematic Decomposition

$$\Theta = 0 \tag{4}$$

$$\dot{u}_\mu = \frac{1}{(1-r^2\,\omega^2)\,z}\,\partial_z - \frac{w^2 r}{1-r^2\,w^2}\,\partial_r \tag{5}$$

$$\omega^\mu = -\frac{r\,w\,z}{1-r^2\,\omega^2}\,\partial_z + \frac{r^2\,\left(\frac{d}{d\,z}\,w\right)\,z + r^2\,w}{(1-r^2\,w^2)\,(2\,r^2\,w^2-2)}\,\partial_r \tag{6}$$

$$\sigma_{z\phi} = \sigma_{\phi z} = -\frac{r^2\,\left(\left(\frac{d}{d\,z}\,w\right)\,z + w\right)}{2z(1-\omega^2 r^2)^{3/2}} \tag{7}$$

Solving $\left(\frac{d}{d\,z}\,w\right)\,z + w = 0$ gives $\omega = \omega_0/z$ where ω_0 is the constant of integration. With this value the shear is eliminated.

3. Raw Data

$$u_\mu = \left[\; -\frac{z}{\sqrt{1-r^2\,w^2}} \quad 0 \quad 0 \quad \frac{r^2\,w}{\sqrt{1-r^2\,w^2}} \;\right] \tag{8}$$

Note $mcs_{abc} = \Gamma^c{}_{ab}$

$$mcs_{1,1,2} = z \tag{9}$$

$$mcs_{1,2,1} = \frac{1}{z} \tag{10}$$

$$mcs_{3,4,4} = \frac{1}{r} \tag{11}$$

$$mcs_{4,4,3} = -r \tag{12}$$

$$\nabla_\mu u^\nu = \begin{bmatrix} 0 & -\frac{r^2\,w\left(\frac{d}{d\,z}\,w\right)}{\sqrt{1-r^2\,w^2}\,(r^2\,w^2-1)\,z} & -\frac{r\,w^2}{\sqrt{1-r^2\,w^2}\,(r^2\,w^2-1)\,z} & 0 \\ \frac{1}{\sqrt{1-r^2\,w^2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{r\,w}{\sqrt{1-r^2\,w^2}} \\ 0 & \frac{\sqrt{1-r^2\,w^2}\left(\frac{d}{d\,z}\,w\right)}{r^4\,w^4-2\,r^2\,w^2+1} & \frac{w\,\sqrt{1-r^2\,w^2}}{r^5\,w^4-2\,r^3\,w^2+r} & 0 \end{bmatrix} \tag{13}$$

$$\nabla_\mu u_\nu = \begin{bmatrix} 0 & \frac{r^2\,w\left(\frac{d}{d\,z}\,w\right)z}{\sqrt{1-r^2\,w^2}\,(r^2\,w^2-1)} & \frac{r\,w^2\,z}{\sqrt{1-r^2\,w^2}\,(r^2\,w^2-1)} & 0 \\ \frac{1}{\sqrt{1-r^2\,w^2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{r\,w}{\sqrt{1-r^2\,w^2}} \\ 0 & \frac{r^2\,\sqrt{1-r^2\,w^2}\left(\frac{d}{d\,z}\,w\right)}{r^4\,w^4-2\,r^2\,w^2+1} & \frac{r\,w\,\sqrt{1-r^2\,w^2}}{r^4\,w^4-2\,r^2\,w^2+1} & 0 \end{bmatrix} \tag{14}$$

$$\dot{u}_\mu u_\nu = \begin{bmatrix} 0 & \frac{1}{\sqrt{1-r^2\,w^2}\,(r^2\,w^2-1)} & -\frac{r\,w^2\,z}{\sqrt{1-r^2\,w^2}\,(r^2\,w^2-1)} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{r^2\,w}{\sqrt{1-r^2\,w^2}\,(r^2\,w^2-1)\,z} & \frac{r^3\,w^3}{\sqrt{1-r^2\,w^2}\,(r^2\,w^2-1)} & 0 \end{bmatrix} \tag{15}$$

$$\dot{u}_\mu u_\nu + \nabla_\mu u_\nu = \begin{bmatrix} 0 & -\frac{\sqrt{1-r^2\,w^2}\left(r^2\,w\left(\frac{d}{d\,z}\,w\right)z+1\right)}{r^4\,w^4-2\,r^2\,w^2+1} & 0 & 0 \\ \frac{1}{\sqrt{1-r^2\,w^2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{r\,w}{\sqrt{1-r^2\,w^2}} \\ 0 & -\frac{r^2\left(\frac{d}{d\,z}\,w\right)z+r^2\,w}{\sqrt{1-r^2\,w^2}\,(r^2\,w^2-1)\,z} & \frac{r\,w}{\sqrt{1-r^2\,w^2}} & 0 \end{bmatrix} \tag{16}$$

3D objects

$$S_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{r^2\left(\frac{d}{d\,z}\,w\right)z+r^2\,w}{\sqrt{1-r^2\,w^2}\,(2\,r^2\,w^2-2)\,z} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{r^2\left(\frac{d}{d\,z}\,w\right)z+r^2\,w}{\sqrt{1-r^2\,w^2}\,(2\,r^2\,w^2-2)\,z} & 0 & 0 \end{bmatrix} \tag{17}$$

$$\omega_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{r^2\left(\frac{d}{d\,z}\,w\right)z+r^2\,w}{\sqrt{1-r^2\,w^2}\,(2\,r^2\,w^2-2)\,z} \\ 0 & 0 & 0 & \frac{r\,w\,\sqrt{1-r^2\,w^2}}{r^2\,w^2-1} \\ 0 & -\frac{r^2\left(\frac{d}{d\,z}\,w\right)z+r^2\,w}{\sqrt{1-r^2\,w^2}\,(2\,r^2\,w^2-2)\,z} & -\frac{r\,w\,\sqrt{1-r^2\,w^2}}{r^2\,w^2-1} & 0 \end{bmatrix} \tag{18}$$

References

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[3] Gabriel Abreu and Matt Visser
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