

the component of momentum flux normal to dA , we multiply by another factor of $\cos\theta$. Integrating, we then obtain

$$p_r(\text{dynes cm}^{-2} \text{ Hz}^{-1}) = \frac{1}{c} \int I_r \cos^2\theta d\Omega. \quad (1.4)$$

Note that F_r and p_r are *moments* (multiplications by powers of $\cos\theta$ and integration over $d\Omega$) of the intensity I_r . Of course, we can always integrate over frequency to obtain the total (integrated) flux and the like.

$$F(\text{erg s}^{-1} \text{ cm}^{-2}) = \int F_r dv \quad (1.5a)$$

$$p(\text{dynes cm}^{-2}) = \int p_r dv \quad (1.5b)$$

$$I(\text{erg s}^{-1} \text{ cm}^{-2} \text{ ster}^{-1}) = \int I_r dv \quad (1.5c)$$

Radiative Energy Density

The specific energy density u_r is defined as the energy per unit volume per unit frequency range. To determine this it is convenient to consider first the energy density per unit solid angle $u_r(\Omega)$ by $dE = u_r(\Omega) dV d\Omega dv$ where dV is a volume element. Consider a cylinder about a ray of length ct (Fig. 1.4). Since the volume of the cylinder is $dAc dt$,

$$dE = u_r(\Omega) dAc dt d\Omega dv.$$

Radiation travels at velocity c , so that in time dt all the radiation in the cylinder will pass out of it:

$$dE = I_r dA d\Omega dt dv.$$

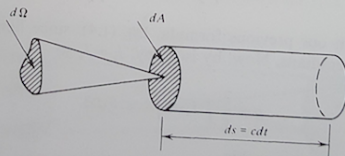


Figure 1.4 Electromagnetic energy in a cylinder.

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Equating the above two expressions yields

$$u_r(\Omega) = \frac{I_r}{c}. \quad (1.6)$$

Integrating over all solid angles we have

$$u_r = \int u_r(\Omega) d\Omega = \frac{1}{c} \int I_r d\Omega,$$

or

$$u_r = \frac{4\pi}{c} J_r, \quad (1.7)$$

where we have defined the *mean intensity* J_r :

$$J_r = \frac{1}{4\pi} \int I_r d\Omega. \quad (1.8)$$

The total radiation density (erg cm^{-3}) is simply obtained by integrating u_r over all frequencies

$$u = \int u_r dv = \frac{4\pi}{c} \int J_r dv. \quad (1.9)$$