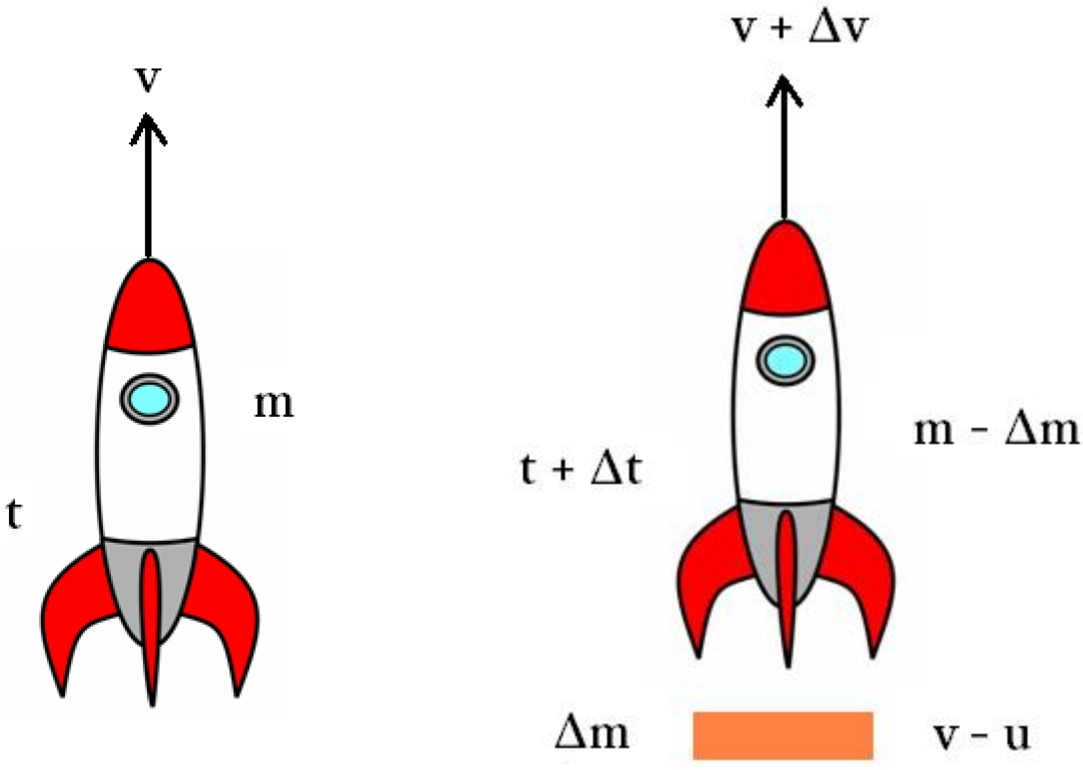
Rocket Equation Derivation using Calculus



- This derivation assumes observations of velocity are from ***your frame of reference*** on the ground watching the rocket launch from the pad.
- We have also excluded gravity for simplicity.
- The rocket starts with a velocity (v) relative to you at time (t) and has a mass (m)
- We then look at the situation of the rocket at a time delta t seconds later.
- The exhaust is ejected at a velocity (u) relative to the rocket. However, we are dealing with a reference frame as viewed from the ground.
- Therefore the velocity will be v – u as viewed from Earth

Derivation

$$P_t = mv$$

$$P_{t+\Delta t} = (m - \Delta m)(v + \Delta v) + \Delta m\,(v - u)$$

Ignore Δm Δv as we can approximate this to be zero

Now expand the brackets out and simplify

$$P_{t+\Delta t} = mv + m\Delta v - u\Delta m$$

$$\Delta P = zero\ (assume\ no\ external\ forces) = m\Delta v - u\Delta m$$

We now take the derivative of this equation

$$\frac{dP}{dt} = 0 = \frac{mdv}{dt} - u\frac{dm}{dt}$$

$$dv = u\frac{dm}{m}$$

$$\int_{v_i}^{v_f} dv = \int_{m_i}^{m_f} u\frac{dm}{m}$$

$$v_f - v_i = u \ln\left(\frac{m_f}{m_i}\right)$$