

Let us search for solitons in the sector with $n = 1$. Asymptotically, as $|\mathbf{x}| \rightarrow \infty$, the scalar field has the form

$$\varphi = e^{i\theta} v. \quad (7.43)$$

For the energy to be finite, the covariant derivative $D_i \varphi$ must decrease more rapidly than $1/r$ (otherwise $\int d^2x |D_i \varphi|^2$ would diverge as $|\mathbf{x}| \rightarrow \infty$). The conventional derivative does not have this property, since

$$\partial_i \varphi = e^{i\theta} v i \partial_i \theta = (e^{i\theta} v) \left(-\frac{i}{r} \varepsilon_{ij} n_j \right),$$

where $n_i = \frac{x_i}{r}$ is a unit vector in the direction \mathbf{x} . Such a slow decrease in