

Let us search for solitons in the sector with  $n = 1$ . Asymptotically, as  $|\mathbf{x}| \rightarrow \infty$ , the scalar field has the form

$$\varphi = e^{i\theta} v. \quad (7.43)$$

For the energy to be finite, the covariant derivative  $D_i\varphi$  must decrease more rapidly than  $1/r$  (otherwise  $\int d^2x |D_i\varphi|^2$  would diverge as  $|\mathbf{x}| \rightarrow \infty$ ). The conventional derivative does not have this property, since

$$\partial_i\varphi = e^{i\theta} v i \partial_i\theta = (e^{i\theta} v) \left( -\frac{i}{r} \varepsilon_{ij} n_j \right),$$

where  $n_i = \frac{x_i}{r}$  is a unit vector in the direction  $\mathbf{x}$ . Such a slow decrease in