Goals

The goal of this paper is to show mathematically how the Sagnac effect does not at all contradict special relativity. In particular, it does not imply a non-constant speed of light. Here we assume the speed of light to be constant, and show that the calculations under this assumption are valid in all reference frames. This paper shows this by measuring the time it takes the receiver to traverse one wavelength of the incoming EM wave, in two reference frames:

1. Frame of reference holding the axis of rotation constant. This is what we think of as the "normal" reference frame. This will generally use the subscript "*ac*" meaning "axis constant".
2. Frame of reference of the receiver. This will generally use the subscript "*sr*" meaning "source/receiver".

This time will then be shown to be exactly related by the Lorentz factor, which is used in special relativity to show time dilation between two inertial reference frames which have mutually relative motion.

Equations Used

Doppler frequency shift: $f_d = \left(\frac{c}{c+v}\right) \cdot f$

Lorentz factor: $y = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

Wavelength: $\lambda = \frac{c}{f}$

Setup Variables

Here we set up some numerical values with which to run the calculations. They are all in reference frame 1.

- Speed of light (constant): $c := 3 \cdot 10^8 = 300.00 \times 10^{+6}$
- Tangential velocity of arm: $v_{ac} := 2 \cdot 10^8 = 200.00 \times 10^{+6}$
- Frequency: $f_{ac} := 10 \cdot 10^9 = 10.00 \times 10^{+9}$

Time Calculation in Reference Frame 1

Since the source is moving in the opposite direction of the wave propagation, we have a doppler shift:

$$f_d := \left(\frac{c}{c+v_{ac}}\right) \cdot f_{ac} = 6.00 \times 10^{+9}$$

Thus, the wavelength becomes:

$$\lambda_{ac} := \frac{c}{f_d} = 50.00 \times 10^{-3}$$

In this frame of reference, the velocity of the arm plus the velocity of the wave is given by $c + v_{ac}$, and the time it takes to traverse one wavelength is:

$$\Delta t_{ac} := \frac{\lambda_{ac}}{c + v_{ac}} = 100.00 \times 10^{-12}$$

Time Calculation in Reference Frame 2

Here there will be no doppler shift, because the source of the wave has no motion in this frame of reference. In other words, it has no motion relative to the arm. However, since frequency is by definition inversely proportional to time, we must account for the time dilation factor, which will serve to make $f_{sr} < f_{ac}$. We must first calculate the Lorentz factor:

$$y := \frac{1}{\sqrt{1 - \frac{v_{ac}^2}{c^2}}} = 600.00 \times 10^{-3} \sqrt{5.00}$$

Since $f \propto \frac{1}{t}$,

$$f_{sr} := y \cdot f_{ac} = 6.00 \times 10^{+9} \sqrt{5.00}$$

And accordingly, the wavelength in this reference frame is given by:

$$\lambda_{sr} := \frac{c}{f_{sr}} = 10.00 \times 10^{-3} \sqrt{5.00}$$

In this frame of reference, according to einstein, the receiver should see the wave coming in at exactly the speed of light, regardless of any motion of the arm relative to the wave. So, the time to traverse one wavelength in this reference frame is given by:

$$\Delta t_{sr} := \frac{\lambda_{sr}}{c} = 33.30 \times 10^{-12} \sqrt{5.00}$$

Conclusions

It can now be shown that this time value Δt_{sr} is related to Δt_{ac} by the Lorentz factor y:

$$y \cdot \Delta t_{sr} = 100.00 \times 10^{-12}$$

Recall that $\Delta t_{ac} = 100.00 \times 10^{-12}$

Thus, it is shown that the time it takes the receiver to traverse a single wavelength of the EM wave in either frame of reference is related by the Lorentz factor, and the speed of light is held constant.