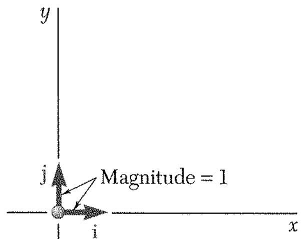


Fig. 2.20

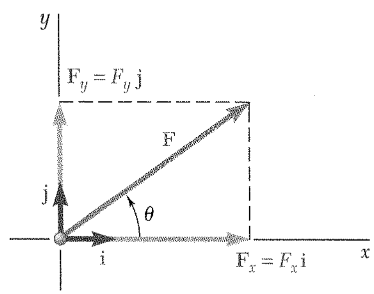
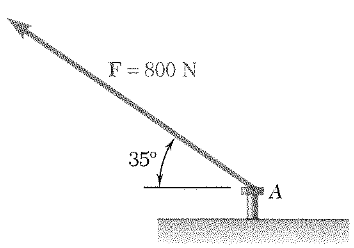
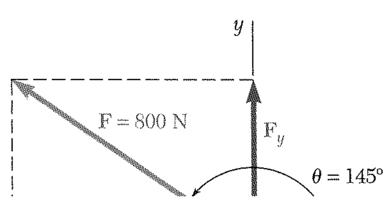


Fig. 2.21



(a)



Two vectors of unit magnitude, directed respectively along positive x and y axes, will be introduced at this point. These ve are called *unit vectors* and are denoted by **i** and **j**, respectively 2.20). Recalling the definition of the product of a scalar and a ve given in Sec. 2.4, we note that the rectangular components $\mathbf{F}_x$ an of a force **F** may be obtained by multiplying respectively the unit tors **i** and **j** by appropriate scalars (Fig. 2.21). We write

$$\mathbf{F}_x = F_x\mathbf{i} \qquad \mathbf{F}_y = F_y\mathbf{j}$$

and

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

While the scalars F_x and F_y may be positive or negative, depen upon the sense of $\mathbf{F}_x$ and of $\mathbf{F}_y$, their absolute values are respect equal to the magnitudes of the component forces $\mathbf{F}_x$ and $\mathbf{F}_y$ scalars F_x and F_y are called the *scalar components* of the forc while the actual component forces $\mathbf{F}_x$ and $\mathbf{F}_y$ should be referr as the *vector components* of **F**. However, when there exists no p bility of confusion, the vector as well as the scalar component **F** may be referred to simply as the *components* of **F**. We note the scalar component F_x is positive when the vector componen has the same sense as the unit vector **i** (that is, the same sense a positive x axis) and is negative when $\mathbf{F}_x$ has the opposite sens similar conclusion may be drawn regarding the sign of the scalar ponent F_y.

Denoting by F the magnitude of the force **F** and by θ the a between **F** and the x axis, measured counterclockwise from the itive x axis (Fig. 2.21), we may express the scalar components of follows:

$$F_x = F\cos\theta \qquad F_y = F\sin\theta$$

We note that the relations obtained hold for any value of the an from 0° to 360° and that they define the signs as well as the abs values of the scalar components F_x and F_y.

Example 1. A force of 800 N is exerted on a bolt A as shown in 2.22*a*. Determine the horizontal and vertical components of the force.

In order to obtain the correct sign for the scalar components F_x an the value 180° − 35° = 145° should be substituted for θ in Eqs. (2.8). ever, it will be found more practical to determine by inspection the sig F_x and F_y (Fig. 2.22*b*) and to use the trigonometric functions of the $\alpha = 35°$. We write, therefore,

$$F_x = -F\cos\alpha = -(800\text{ N})\cos 35° = -655\text{ N}$$
$$F_y = +F\sin\alpha = +(800\text{ N})\sin 35° = +459\text{ N}$$

The vector components of **F** are thus

Example 2. A man pulls with a force of 300 N on a rope attached t a building, as shown in Fig. 2.23*a*. What are the horizontal and vertical com ponents of the force exerted by the rope at point A?

It is seen from Fig. 2.23*b* that

$$F_x = +(300\text{ N})\cos\alpha \qquad F_y = -(300\text{ N})\sin\alpha$$

Observing that AB = 10 m, we find from Fig. 2.23*a*

$$\cos\alpha = \frac{8\text{ m}}{AB} = \frac{8\text{ m}}{10\text{ m}} = \frac{4}{5} \qquad \sin\alpha = \frac{6\text{ m}}{AB} = \frac{6\text{ m}}{10\text{ m}} = \frac{3}{5}$$

We thus obtain

$$F_x = +(300\text{ N})\tfrac{4}{5} = +240\text{ N} \qquad F_y = -(300\text{ N})\tfrac{3}{5} = -180\text{ N}$$

and write

$$\mathbf{F} = (240\text{ N})\mathbf{i} - (180\text{ N})\mathbf{j}$$

When a force **F** is defined by its rectangular components F_x an F_y (see Fig. 2.21), the angle θ defining its direction can be obtaine by writing

$$\tan\theta = \frac{F_y}{F_x} \qquad (2.9$$

The magnitude F of the force can be obtained by applying th Pythagorean theorem and writing

$$F = \sqrt{F_x^2 + F_y^2} \qquad (2.10$$

or by solving for F one of the Eqs. (2.8).

Example 3. A force $\mathbf{F}$ = (700 lb)**i** + (1500 lb)**j** is applied to a bolt A Determine the magnitude of the force and the angle θ it forms with th horizontal.

First we draw a diagram showing the two rectangular components of th force and the angle θ (Fig. 2.24). From Eq. (2.9), we write

$$\tan\theta = \frac{F_y}{F_x} = \frac{1500\text{ lb}}{700\text{ lb}}$$

Using a calculator,† we enter 1500 lb and divide by 700 lb; computin the arc tangent of the quotient, we obtain θ = 65.0°. Solving the second o Eqs. (2.8) for F, we have

$$F = \frac{F_y}{\sin\theta} = \frac{1500\text{ lb}}{\sin 65.0°} = 1655\text{ lb}$$

The last calculation is facilitated if the value of F_y is stored when originall entered; it may then be recalled to be divided by $\sin\theta$.