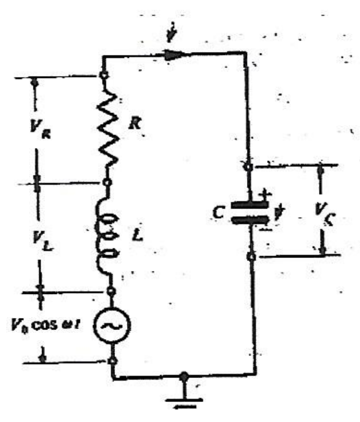
PHYS367 chapter 4 homework set
Fall 2017

In-class part. [50 points]

1. A system with $m = 0.010$ kg, $s = 36$ Nm^{-1} and $b = 0.50$ kgs^{-1} is driven by a harmonically varying fore of amplitude 3.6 N. Find the amplitude A and the phase constant ϕ of the steady-state motion when the angular frequency is (a) 8.0 s^{-1}, (b) 80 s^{-1}, and (c) 800 s^{-1}.
2. Show that the displacement amplitude A is a miximum at the driving frequency given by $\omega^2 = \omega_0^2 - \frac{1}{2}\gamma^2$.
3. Show that the acceleration amplitude $\omega^2 A$ is a maximum at the driving frequency given by $\omega^2 = \omega_0^2 / (1 - \gamma^2 / 2\omega_0^2) \approx \omega_0^2 + \frac{1}{2}\gamma^2$
where the approximation is good when the damping is very light.
4. Show that, for driving force frequencies close to resonance ($\omega \approx \omega_0$) in a very lightly dampled system, the response function may be approximated by the Lorenzian
$$L(\omega) \equiv \frac{\frac{1}{4}\gamma^2}{(\omega_0 - \omega)^2 + \frac{1}{4}\gamma^2}$$
5. By finding the value of ω which makes the impedance $|Z(\omega)|$ a minimum, confirm that the velocity amplitude is a maximum when $\omega = \omega_0$.
6. For motion $\psi = (7.5\text{mm})\cos\left[(6.28s^{-1})t + 27°\right] - (2.3\text{mm})\sin\left[(6.20s^{-1})t + 121°\right]$, find (a) the frequency (Hz), and (b) the time interval separatingsuccessive beats. Be aware the eqution expressed in milimeter and degree!
7. A single driving force produces steady-state forced vibrations with amplitude 10 mm. A second driving force of the same frequency as the first, acting on the same system, produces a steady-state amplitude of 20 mm. When both forces are acting simultaneously, the steady-state amplitude is 15 mm. What is the phase difference between the two forces?
8. The *LCR* circuit with a driving force $V_0\cos\omega t$.
   a. Using the table below, express the equation of motion using L, C, R and driving (external) force $V_0\cos\omega t$. [5 points]
   b. Derive impedance, $Z(\omega)$, at the resonant frequency ($\omega = \omega_0$). [5 points]



$V_0 \cos\omega t = \ddot{\psi} + \frac{R}{L}\dot{\psi} + \frac{1}{LC}\psi$

$Z = \frac{1}{i\omega} K(\omega)$ $\quad K(\omega) = \frac{1}{(s - m\omega^2) + ib\omega}$