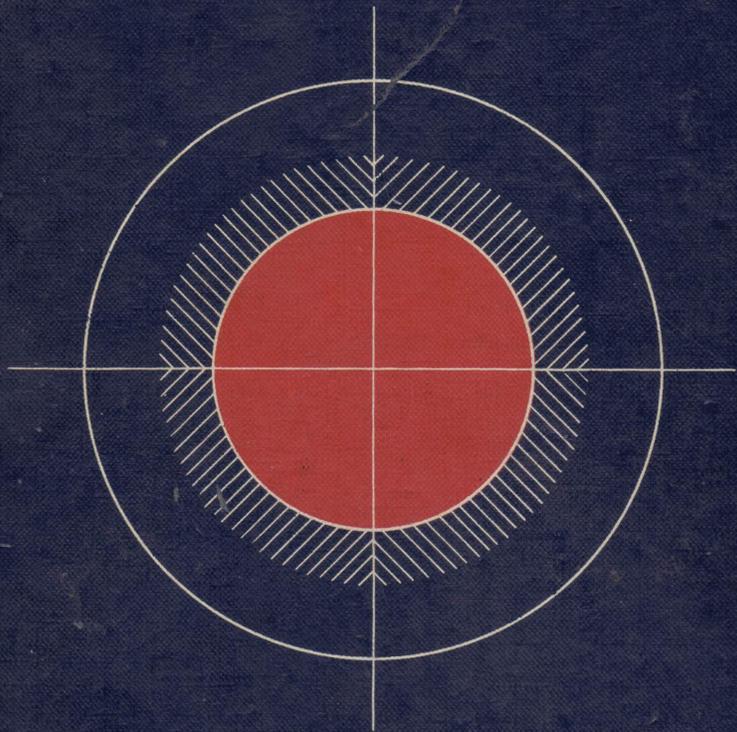


Gene F. Franklin and J. David Powell

**DIGITAL
CONTROL
OF DYNAMIC SYSTEMS**



6 / Design of Digital Control Systems Using State-Space Methods

6.1 INTRODUCTION

In Chapter 5, we discussed how to design digital controllers using transform techniques, methods now commonly designated as "classical design." The goal of this chapter is to solve the identical problem using different techniques which are based on the state-space or modern control formulation. The difference in the two approaches is entirely in the design method since the end result, a set of difference equations providing control, is identical. Advantages of modern control are especially apparent when engineers design controllers for systems with more than one control input or sensed output; however, to illustrate the ideas of state-space design, we will devote our efforts in this chapter to single input/output systems. Techniques for the multi-input–multi-output design are discussed in Chapter 9.

6.2 SYSTEM REPRESENTATION

A continuous, linear, constant-coefficient system of differential equations can always be expressed as a set of first-order matrix differential equations¹:

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{G}u + \mathbf{G}_1w, \quad (6.1)$$

where u is the control input to the system, and w is a disturbance input. The output can be expressed as a linear combination of the state, \mathbf{x} , and the input as

$$y = \mathbf{H}\mathbf{x} + Ju. \quad (6.2)$$

The representations (6.1) and (6.2) are not unique. Given one state representation, any nonsingular linear transformation of that state $\boldsymbol{\xi} = \mathbf{T}\mathbf{x}$ is also an allow-

¹ We assume the reader has some knowledge of matrices. The results we require and references to study material are given in Appendix C. To distinguish vectors and matrices, we will use bold-face type.