

3. Consider the function of two variables

$$u(x, y) = f(x - y) + g\left(x + \frac{1}{3}y\right),$$

where $f(s)$ and $g(t)$ are each arbitrary functions of a single variable.

Using the change of variables

$$s = x - y,$$

$$t = x + \frac{1}{3}y,$$

use the chain rule to determine the first and second derivatives of u with respect to x and y in terms of derivatives of f and g .

Hence, show that the second derivatives satisfy

$$u_{xx} = 2u_{xy} + 3u_{yy} \quad \text{where} \quad u_{xx} = \partial^2 u / \partial x^2 \text{ etc.}$$

4. If $z = \frac{-4}{1 + i\sqrt{3}},$

- (i) find the real and imaginary part of z ;
- (ii) find the modulus and argument of z ;
- (iii) find the modulus and argument of z^2 ;
- (iv) find the moduli and arguments of all values of $z^{1/3}$;
- (v) plot the results of (i) - (iv) on a rough sketch of the complex plane.

Quote arguments within the range $0 \leq \theta < 2\pi$ in both degrees and radians.