

$$3. \quad u(x,y) = f(x-y) + g(x + \frac{1}{3}y) \quad s = x-y \quad t = x + \frac{1}{3}y$$

$$du = \frac{\partial u}{\partial f} df + \frac{\partial u}{\partial g} dg$$

PAGE 1

$$df = \frac{df}{ds} ds \Rightarrow$$

$$dg = \frac{dg}{dt} dt \Rightarrow$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial s} \frac{\partial s}{\partial x} = \frac{\partial f}{\partial s}$$

$$\frac{\partial g}{\partial x} = \frac{\partial g}{\partial t} \frac{\partial t}{\partial x} = \frac{\partial g}{\partial t}$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial s} \frac{\partial s}{\partial y} = -\frac{\partial f}{\partial s}$$

$$\frac{\partial g}{\partial y} = \frac{\partial g}{\partial t} \frac{\partial t}{\partial y} = \frac{1}{3} \frac{\partial g}{\partial t}$$

$$u_x = \frac{\partial u}{\partial x} = \frac{\partial u}{\partial f} \frac{\partial f}{\partial s} + \frac{\partial u}{\partial g} \frac{\partial g}{\partial t} = u_x$$

$$u_y = -\frac{\partial u}{\partial f} \frac{\partial f}{\partial s} + \frac{1}{3} \frac{\partial u}{\partial g} \frac{\partial g}{\partial t} = u_y$$

$$u_{xx} = \left(\frac{\partial f}{\partial s} \frac{\partial}{\partial f} + \frac{\partial g}{\partial t} \frac{\partial}{\partial g} \right) \left(\frac{\partial f}{\partial s} \frac{\partial u}{\partial s} + \frac{\partial g}{\partial t} \frac{\partial u}{\partial g} \right)$$

$$= \left(\frac{\partial f}{\partial s} \right)^2 u_{ff} + \left(\frac{\partial g}{\partial t} \right)^2 u_{gg} + \left(\frac{\partial f}{\partial s} \frac{\partial g}{\partial t} \right) u_{fg}$$

$$+ \left(\frac{\partial f}{\partial s} \frac{\partial g}{\partial t} \right) u_{gf}$$

$$u_{gf} = u_{fg}$$

$$\text{so } u_{xx} = \left(\frac{\partial f}{\partial s} \right)^2 u_{ff} + \left(\frac{\partial g}{\partial t} \right)^2 u_{gg} + 2 \left(\frac{\partial f}{\partial s} \frac{\partial g}{\partial t} \right) u_{fg}$$