Parallel Transport Examined

Parallel Transport equations for a latitude[$\theta = Const$]. We hold r=constant

$$\frac{dA^{\theta}}{d\varphi} - \cos\theta \sin\theta A^{\varphi} = 0$$

$$\frac{dA^{\varphi}}{d\varphi} + \frac{1}{r} A^{r} + \cot\theta A^{\theta} = 0$$

$$\frac{dA^{r}}{d\varphi} - (r-2M) A^{\varphi} = 0$$

$$\frac{dA^{t}}{d\varphi} = 0$$

Solutions:

$$A^{\varphi} = C_1 \cos k\varphi + C_2 \sin k\varphi$$

$$A^{\theta} = \frac{\sin\theta \cos\theta}{k}[C_1 \sin k\varphi - C_2 \cos k\varphi] + C_3$$

$$A^{r} = \frac{r-2M}{k}[C_1 \sin k\varphi - C_2 \cos k\varphi] - C_3 r \cos^2\theta$$

$$A^{t} = C_4$$

In the above calculations $k^2 = \frac{r-2M}{r} Sin^2\theta + \cos^2\theta$

Observations:

1) Even if A^{r} is kept zero at the initial point of the round tour, it assumes non zero values in the path in a general manner. The vector seems to lift out of the tangent plane.

2) We cannot assign integral values to k and so we do nothave periodic functions of 2π in the trigonometric functions. Nevertheless the lifting out of the vector from the tangent plane should be given attention

3) With A^{t} we have no problem

For transport along a meridian [$\varphi = Cons\tan t, r = const$], the equations are:

$$\frac{dA^{\theta}}{d\theta} + \frac{1}{r} A^{r} = 0$$

$$\frac{dA^{\varphi}}{d\theta} + \frac{1}{r} A^{r} + \cot\theta\, A^{\varphi} = 0$$

$$\frac{dA^{r}}{d\theta} - (r - 2M) A^{\theta} = 0$$

$$\frac{dA^{t}}{d\theta} = 0$$

Solutions:

$$A^{\theta} = C_1 Cos k\theta + C_2 Sin k\theta$$

$$A^{r} = \frac{r-2M}{k}[C_1 Sin\, k\theta - C_2 Cos\, k\theta] + C_3$$

$$A^{\varphi} Sin\theta = -\frac{r-2M}{2kr}[C_1(\frac{Sin(1-k)\theta}{1-k} - \frac{Sin(1+k)\theta}{1+k}) + C_2(\frac{Cos(1+k)\theta}{1+k} + \frac{Sin(1-k)\theta}{1-k})] + C_3 / r + C'$$

$$A^{t} = C_4$$

$$[k^2 = \frac{r-2M}{r}]$$

In the third equation above by substituting $\theta = 0$ on either side we can reduce the number of constants. Therefore we have four arbitrary constants for the four equations.

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