Voltage, as that term is commonly used, is the line integral of $\mathbf{E}_s$. Voltage was defined with this meaning in equation 133, which should now be written more explicitly as

$$V = \int \mathbf{E}_s \cdot ds \qquad [179]$$

Electromotive force is a similar line integral of $\mathbf{E}_m$:

$$\text{Electromotive force} = \int \mathbf{E}_m \cdot ds \qquad [180]$$

As mentioned above, electromotive force may result from magnetic action, as in a generator or a transformer. It may also come from

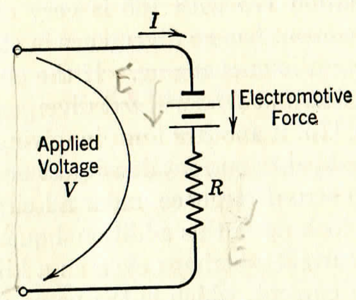

FIG. 33

chemical action in a battery, or from heat in a thermocouple, or from various other physical processes. In general, an electromotive force appears when energy of some other kind, such as chemical energy, or heat energy, or mechanical energy, is changed into electric energy.

Ohm's law is written in equation 159 for a section of circuit in which there is no electromotive force, and current flows as a result of applied voltage only:

$$IR = V = \int \mathbf{E}_s \cdot ds \qquad [181]$$

But, in a part of a circuit where there is electromotive force, the electromotive force will contribute to the flow of current, and

$$IR = V + \text{Electromotive force} \qquad [182]$$

Thus, in the circuit of Fig. 33, the applied voltage is V, the resistance is R, and the electromotive force of a battery assists the flow of current. Current is therefore

$$I = \frac{V + \text{Electromotive force}}{R} \qquad [183]$$

as in equation 182. Equation 182 may also be written

$$IR = \int \mathbf{E}_s \cdot ds + \int \mathbf{E}_m \cdot ds = \int \mathbf{E} \cdot ds \qquad [184]$$

Voltage in Fig. 33 is mathematically the line integral of $\mathbf{E}_s$ between the upper and lower terminals, following any desired path of integration. These terminals and the wires connected to them are, of course, charged bodies. If the path of integration is so chosen that it does not pass through any region in which any electromotive force exists, so that $\mathbf{E}_m$ is zero, the voltage is equally well expressed as the line integral of $\mathbf{E}$, as may be seen from equation 184, for, in the absence of electromotive force, $\mathbf{E} = \mathbf{E}_s$.

A particularly interesting special case appears in a closed circuit containing electromotive force. If the section of circuit under consideration in equations 182 and 184 is expanded until it becomes the whole circuit, and if the two terminal points approach each other until they coincide, the paths of integration of the line integrals become closed paths. But the line integral of $\mathbf{E}_s$ about any closed path is zero (equation 4) and equation 184 thus reduces, for a closed circuit, to

$$IR = \text{Electromotive force} = \oint \mathbf{E} \cdot ds = \oint \vec{E}_m \cdot d\vec{s} \qquad [185]$$

This relation will be of use in the next chapter, in exploring the magnetic field.

It may be noted in passing that the term that appears in ordinary alternating-current circuit computations as "inductive reactance voltage drop" is, in equations 182 and 184, part of the electromotive force term, for it is fundamentally a "counter electromotive force" produced by changing magnetic field. If there is a condenser in a circuit, its voltage is part of the $\int \mathbf{E}_s \cdot ds$ term.

PROBLEMS

1. Find, from tables or otherwise, the conductivity of silver, gold, copper, iron, and aluminum. Give values in mhos per meter. What is the mks unit of resistivity?

2. Derive Kirchhoff's first law, as stated on page 71, from equation 166. Kirchhoff's second law says, in effect, that, in the absence of electromotive forces, the sum