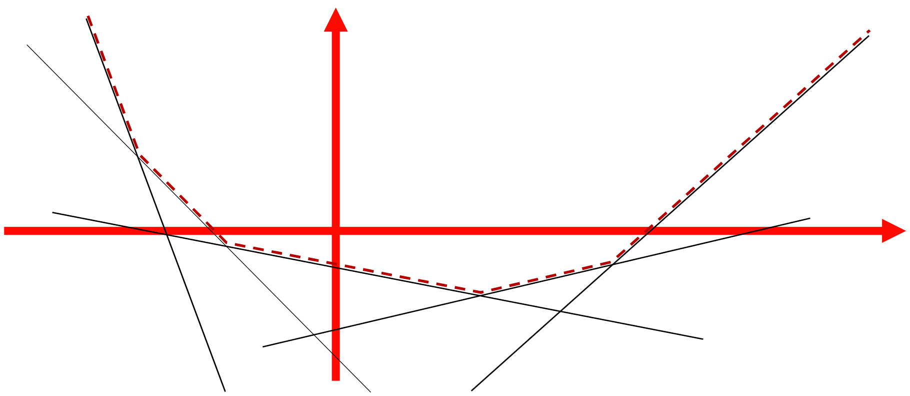
Objective: maximum of linear functions



$$\min_{x \in \mathbf{R}^m} \left(\max_{1 \le i \le N} c_i^T x + d_i \right)$$
$$\text{subject to } Ax \le b$$

$$\min_{x \in \mathbf{R}^m} \left(\begin{array}{l} \min_{t \in \mathbf{R}} \; t \\ \quad \text{subject to } c_i^T x + d_i \le t \quad \forall i \end{array} \right)$$
$$\text{subject to } Ax \le b$$

$$\min_{x \in \mathbf{R}^m, t \in \mathbf{R}} \; t$$
$$\text{subject to } \begin{bmatrix} A & 0_n \\ c_1^T & -1 \\ c_2^T & -1 \\ \vdots & \vdots \\ c_N^T & -1 \end{bmatrix} \begin{bmatrix} x \\ t \end{bmatrix} \le \begin{bmatrix} b \\ -d_1 \\ -d_2 \\ \vdots \\ -d_N \end{bmatrix}$$

$$t = [0_m^T \; 1] \begin{bmatrix} x \\ t \end{bmatrix}$$

$$\boxed{\begin{array}{l} \min_{z \in \mathbf{R}^{\tilde{m}}} \; \tilde{c}^T z \\ \quad \text{subject to } \tilde{A} z \le \tilde{b} \end{array}}$$

aside: an alternative way to look at *max of absolute values*

Given a collection of numbers,

$$\{z_1, z_2, \ldots, z_N\}$$

Determining the maximum entry of the absolute values

$$\max_{1 \le i \le N} |z_i|$$

can be recast as an LP:

find the smallest number *t* which has

$$z_1 \le t, \ldots, z_N \le t.$$
$$z_1 \ge -t, \ldots, z_N \ge -t.$$

Hence

$$\max_{1 \le i \le N} |z_i| = \begin{array}{l} \min_{t \in \mathbf{R}} \ t \\ \quad \text{subject to } z_i \le t \quad i = 1, 2, \ldots, N \\ \qquad\qquad\quad -z_i \le t \quad i = 1, 2, \ldots, N \end{array}$$