aside: an alternative way to look at *sum of abs*

The sum of the absolute values a collection of numbers, $\{z_1, z_2, \ldots, z_N\}$

$$f(z) = \sum_{i=1}^{N} |z_i|$$

can rewritten as the solution to an LP:

find $t_1, t_2, \cdots t_N$, with minimum sum, subject to:

$$\begin{gathered} -t_1 \le z_1 \le t_1 \\ \vdots \\ -t_N \le z_N \le t_N \end{gathered}$$

Hence

$$\sum_{i=1}^{N} |z_i| = \begin{array}{ll} \min\limits_{t \in \mathbf{R}^N} & \sum\limits_{i=1}^{N} t_i \\ \text{subject to} & \begin{array}{rl} z_i \le t_i & i = 1, 2, \ldots, N \\ -z_i \le t_i & i = 1, 2, \ldots, N \end{array} \end{array}$$

Objective: sum of absolute value of linear functions

$$\min_{x\in\mathbf{R}^m} \sum_{i=1}^{N} |c_i^T x + d_i| \quad \text{subject to } Ax \le b$$

$$\min_{x\in\mathbf{R}^m} \left(\begin{array}{l} \displaystyle\min_{t\in\mathbf{R}^N} \sum_{i=1}^{N} t_i \\ \text{subject to } c_i^T x + d_i \le t_i \quad \forall i \\ \qquad\qquad\quad -c_i^T x - d_i \le t_i \quad \forall i \end{array} \right)$$

$$\text{subject to } Ax \le b$$

$$C = \begin{bmatrix} c_1^T \\ c_2^T \\ \vdots \\ c_N^T \end{bmatrix}$$

$$\min_{x\in\mathbf{R}^m, t\in\mathbf{R}^N} \sum_{i=1}^{N} t_i$$

$$\text{subject to } \begin{bmatrix} A & 0_{m\times N} \\ C & -I_N \\ -C & -I_N \end{bmatrix} \begin{bmatrix} x \\ t \end{bmatrix} \le \begin{bmatrix} b \\ -d \\ d \end{bmatrix}$$

$$\sum_{i=1}^{N} t_i = [0_m^T \ \mathbf{1}_N^T] \begin{bmatrix} x \\ t \end{bmatrix}$$

$$\boxed{\begin{array}{l} \displaystyle\min_{z\in\mathbf{R}^{\tilde m}} \tilde c^T z \\ \text{subject to } \tilde A z \le \tilde b \end{array}}$$