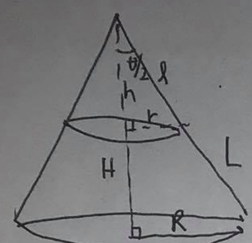

$\tan\frac{\theta}{2} = \frac{r}{h}$

$h\tan\frac{\theta}{2} = r$

$\tan\frac{\theta}{2} = \frac{R}{H}$

$H\tan\frac{\theta}{2} = R$

let $k = \tan\frac{\theta}{2}$

so $r = hk$ and $R = Hk$

$l = \sqrt{h^2 + r^2}$

$l = \sqrt{h^2 + (hk)^2}$

$l = \sqrt{h^2 + h^2k^2}$

$\boxed{l = h\sqrt{1+k^2}}$

$\boxed{L = H\sqrt{1+k^2}}$

$\frac{\partial l}{\partial h} = \sqrt{1+k^2}$

$\frac{\partial l}{\partial k} = \frac{1}{2}h(1+k^2)^{-\frac{1}{2}}(2k)$

$\frac{\partial l}{\partial k} = \frac{kh}{\sqrt{1+k^2}}$

$\frac{\partial L}{\partial H} = \sqrt{1+k^2} = \frac{\partial l}{\partial h}$

$\frac{\partial L}{\partial k} = \frac{1}{2}H(1+k^2)^{-\frac{1}{2}}(2k)$

$\frac{\partial L}{\partial k} = \frac{Hk}{\sqrt{1+k^2}}$

$V_{frustum} = V^{big}_{CONE} - V^{small}_{cone}$

$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left[R^2H - r^2h\right]$

$= \frac{1}{3}\pi\left[(Hk)^2H - (hk)^2h\right]$

$\boxed{V_{frustum} = \frac{1}{3}\pi k^2\left[H^3 - h^3\right]}$

Constraint

$TSA_{frustum} = TSA^{big}_{CONE} - LSA^{small}_{cone} + Base^{small}_{cone}$

$= (\pi R^2 + \pi RL) - \pi rl + \pi r^2$

$= \pi(Hk)^2 + \pi(Hk)L - \pi hkl + \pi(hk)^2$

$\boxed{TSA_{frustum} = \pi\left[H^2k^2 + HkL - hkl + h^2k^2\right]}$

Objective Function

Partial Der. wrt H : $TSA_H = \lambda V_H$

$2Hk^2 + Hk\frac{\partial L}{\partial H} + Lk = \frac{1}{3}\lambda k^2\left[3H^2\right]$

$2Hk^2 + Hk\frac{\partial L}{\partial H} + Lk = \lambda k^2H^2$

$2Hk^2 + Hk\sqrt{1+k^2} + H\sqrt{1+k^2}\,k = \lambda k^2H^2$

$2k + 2\sqrt{1+k^2} = \lambda kH$ (Equation 1)

Partial Der. wrt h: $\quad TSA_h = \lambda V_h$

$$-hk\frac{\partial l}{\partial h} - lk + 2hk^2 = \frac{1}{3}\lambda k^2\left[-3h^2\right]$$

$$-hk\sqrt{1+k^2} - hk\sqrt{1+k^2} + 2hk^2 = -\lambda k^2 h^2$$

$$-2\sqrt{1+k^2} + 2k = -\lambda kh \qquad \text{(Equation 2)}$$

Partial Derivative wrt k: $\quad TSA_k = \lambda V_k$

$$2H^2k + Hk\frac{\partial L}{\partial k} + LH - hk\frac{\partial l}{\partial k} - lh + 2h^2k = \frac{2}{3}\lambda k\left[H^3 - h^3\right]$$

$$2H^2k + Hk\left(\frac{Hk}{\sqrt{1+k^2}}\right) + H^2\sqrt{1+k^2} - hk\left(\frac{kh}{\sqrt{1+k^2}}\right) - h^2\sqrt{1+k^2} + 2h^2k = \frac{2}{3}\lambda k\left[H^3 - h^3\right]$$

$$2H^2k + \frac{H^2k^2}{\sqrt{1+k^2}} + H^2\sqrt{1+k^2} - \frac{h^2k^2}{\sqrt{1+k^2}} - h^2\sqrt{1+k^2} + 2h^2k = \frac{2}{3}\lambda k\left[H^3 - h^3\right]$$

(Equation 3)

[redacted] $\quad V = C$

$$\frac{1}{3}\pi k^2\left[H^3 - h^3\right] = C \qquad \text{Equation (4)}$$