

Solution

Question 3

a)

To model the system we must first derive an equation to find the torque generated by the turbine blades as this will be the **Input torque**:

Multiply the Mass by the radius to give the 1st moment of mass = rm
Multiply again by the radius to get the 2nd moment of mass = r^2m
This is the moment of inertia I

$$Ta = I. \alpha \text{ where } \alpha \text{ is the angular acceleration.}$$

Angular acceleration is given as

$$\alpha = \frac{\Delta\omega}{\Delta\text{time}}$$

Angular velocity ω will be a tangential product of the wind pressure P acting on the turbine blades.

The power generated will be given by:

$$\text{Power} = \text{Torque} \cdot \omega$$

Now consider the input torque T acting on section L1 and L2 and L3:

*“Subjected to an applied torque T , the power transmitted will have the following conditions:
Equilibrium: Applied torque $T = Ta = Tb$ i.e the torque is the same in each section of the shaft.*

So the applied torque T acts on sections L1 and L2, Begin by finding the polar second moment of area for each section L1-L3”:

$$J = \frac{\pi D^4}{32}$$

The max shear stress is found acting at the outer surface of each Radii (1-3) , in other words at the R_{max} position. This stress is found with the torsion equation:

$$\tau_{L1} = \frac{TR_1}{J_1}$$

$$\tau_{L2} = \frac{TR_2}{J_3}$$

$$\tau_{L1} = \frac{TR_1}{J_3}$$

(For all three the input torque T will be the same amount)

It stands to reason therefore that the greatest shear stress will be found at L3, the smallest shaft.

“Using the torsion equation and noting that the max stress τ occurs at the maximum radius, i.e at the outer surface”:

$$\frac{T}{J} = \frac{\tau}{R} \rightarrow T_{\text{max}} = \frac{\tau J}{R}$$

So therefore $Power = \frac{vJ}{R} \cdot \omega$

Apply this formula in sequence for each section L1-L3 to find the final power for section L3.

Now account for the armature housing which produces a dampening effect of constant **C**:
The resistive torque will be known as T_r :

$$\text{Net power transmission} = \left(\frac{vJ}{R} - T_r\right) \cdot \omega$$