Problem 3

APEX [redacted]

Project Name ______________
Project No. ________ By ______ Checked ______
Date ______________ Sheet ____ of ____

$\vec{A} = \hat{i}\, xy + \hat{j}\,(2y - z^2) + \hat{k}\, xz$

Transform Eq. $x = r\cos\Phi \sin\varphi$
$y = r\sin\Phi \sin\varphi$
$z = r\cos\Phi$

using results from previous works

$$\begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix} = \underbrace{\begin{bmatrix} \sin\Phi\cos\varphi & \cos\Phi\cos\varphi & -\sin\Phi \\ \sin\Phi\sin\varphi & \cos\Phi\sin\varphi & \cos\Phi \\ \cos\Phi & -\sin\Phi & 0 \end{bmatrix}}_{M} \begin{bmatrix} \hat{r} \\ \hat{\Phi} \\ \hat{\varphi} \end{bmatrix}$$

therefore: $\hat{i} = \sin\Phi\cos\varphi\,\hat{r} + \cos\Phi\cos\varphi\,\hat{\Phi} - \sin\Phi\,\hat{\varphi}$
$\hat{j} = \sin\Phi\sin\varphi\,\hat{r} + \cos\Phi\sin\varphi\,\hat{\Phi} + \cos\Phi\,\hat{\varphi}$
$\hat{k} = \cos\Phi\,\hat{r} - \sin\Phi\,\hat{\Phi}$

we can now plug everything in and solve accordingly.
pluging in for $x, y, \&\ z$ first

$\vec{A} = \hat{i}\,(r\cos\Phi\sin\varphi)(r\sin\Phi\sin\varphi) + \hat{j}\left[(2r\sin\Phi\sin\varphi) - r^2\cos^2\Phi\right] + \hat{k}\,(r\cos\Phi\sin\varphi)(r\cos\Phi)$

$A = \hat{i}\,(r^2\cos\Phi\sin\Phi\sin^2\varphi) + \hat{j}\left[2r\sin\Phi\sin\varphi - r\cos^2\Phi\right] + \hat{k}\,(r^2\cos^2\Phi\sin\varphi)$

Adding in unit vector transforms:

$A = (r^2\cos\Phi\sin\Phi\sin^2\varphi)(\sin\Phi\cos\varphi\,\hat{r} + \cos\Phi\cos\varphi\,\hat{\Phi} - \sin\Phi\,\hat{\varphi})$
$\left[= r^2\cos\Phi\sin^2\Phi\sin^2\varphi\cos\varphi\,\hat{r} + r^2\cos^2\Phi\sin\Phi\sin^2\varphi\cos\varphi\,\hat{\Phi} - r^2\cos\Phi\sin^2\Phi\sin^2\varphi\,\hat{\varphi}\right]$ 1st term

$(2r\sin\Phi\sin\varphi - r\cos^2\Phi)(\sin\Phi\sin\varphi\,\hat{r} + \cos\Phi\sin\varphi\,\hat{\Phi} + \cos\Phi\,\hat{\varphi})$ 2nd term
$= \left[(2r\sin^2\Phi\sin^2\varphi\,\hat{r} - r\sin\Phi\sin\varphi\cos^2\Phi\,\hat{r}) + (2r\sin\Phi\cos\Phi\sin^2\varphi\,\hat{\Phi} - r\cos^3\Phi\sin\varphi\,\hat{\Phi}) + (2r\sin\Phi\sin\varphi\cos\varphi\,\hat{\varphi} - r\cos^3\Phi\,\hat{\varphi})\right]$

$(r^2\cos^2\Phi\sin\varphi)(\cos\Phi\,\hat{r} - \sin\Phi\,\hat{\Phi}) = \left[r^2\cos^3\Phi\sin\varphi\,\hat{r} - r^2\cos^2\Phi\sin\Phi\sin\varphi\,\hat{\Phi}\right]$

$\hat{r}$ terms:

$r^2\cos\Phi\sin^2\Phi\sin^2\varphi\cos\varphi\,\hat{r} + 2r\sin^2\Phi\sin^2\varphi\,\hat{r} - r\sin\Phi\sin\varphi\cos^2\Phi\,\hat{r} + r^2\cos^3\Phi\sin\varphi\,\hat{r}$

$\boxed{A_r = (r^2\cos\Phi\sin^2\Phi\sin^2\varphi\cos\varphi + 2r\sin^2\Phi\sin^2\varphi - r\sin\Phi\sin\varphi\cos^2\Phi + r^2\sin\varphi\cos^3\Phi)}$

$\hat{\Phi}$ terms:

$r^2\cos^2\Phi\sin\Phi\sin^2\varphi\cos\varphi\,\hat{\Phi} + 2r\sin\Phi\cos\Phi\sin^2\varphi\,\hat{\Phi} - r\cos^3\Phi\sin\varphi\,\hat{\Phi} - r^2\cos^2\Phi\sin\Phi\sin\varphi\,\hat{\Phi}$

$\boxed{A_\Phi = (r^2\cos^2\Phi\sin^2\Phi\sin^2\varphi\cos\varphi + 2r\sin\Phi\cos\Phi\sin^2\varphi - r\cos^3\Phi\sin\varphi - r^2\cos^2\Phi\sin\Phi\sin\varphi)}$

$\hat{\varphi}$ terms: $-r^2\cos\Phi\sin^2\Phi\sin^2\varphi\,\hat{\varphi} + 2r\sin\Phi\sin\varphi\cos\varphi\,\hat{\varphi} - r\cos^3\Phi\,\hat{\varphi} + 0$

$\boxed{A_\varphi = (-r^2\cos\Phi\sin^2\Phi\sin^2\varphi + 2r\sin\Phi\sin\varphi\cos\varphi - r\cos^3\Phi)}$