This print-out should have 8 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

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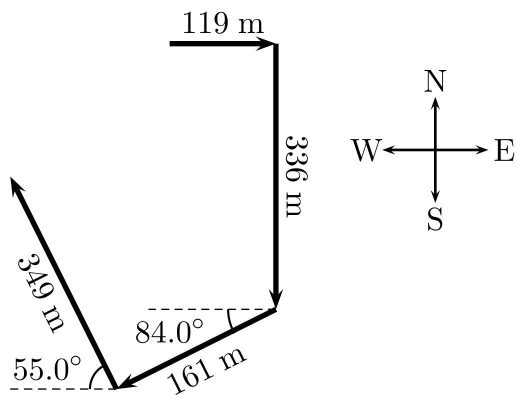
001 (part 1 of 2) 10.0 points

A person walks the path shown. The total trip consists of four straight-line paths.

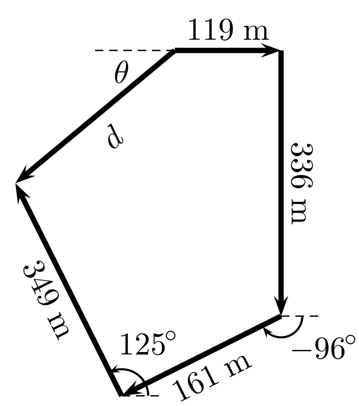


Note: Figure is not drawn to scale.

At the end of the walk, what is the magnitude of the person's resultant displacement measured from the starting point?

Correct answer: 231.956 m.

Explanation:



Note: Figure is not drawn to scale.

$$\text{Let}: \quad \begin{aligned} d_1 &= 119.0 \text{ m} \\ \theta_1 &= 0^\circ \\ d_2 &= 336.0 \text{ m} \\ \theta_2 &= -90^\circ \\ d_3 &= 161.0 \text{ m} \\ \theta_3 &= 84.0^\circ - 180.0^\circ = -96^\circ \\ d_4 &= 349.0 \text{ m} \end{aligned}$$

$$\theta_4 = 180.0^\circ - 55.0^\circ = 125^\circ$$

$$\Delta x_1 = (119 \text{ m}) \cos 0^\circ = 119 \text{ m}$$

$$\Delta y_2 = (336 \text{ m}) \cos(-90^\circ) = -336 \text{ m}$$

$$\begin{aligned} \Delta x_3 &= (161 \text{ m}) \cos(-96^\circ) \\ &= -16.8291 \text{ m} \end{aligned}$$

$$\begin{aligned} \Delta y_3 &= (161 \text{ m}) \sin(-96^\circ) \\ &= -160.118 \text{ m} \end{aligned}$$

$$\begin{aligned} \Delta x_4 &= (349 \text{ m}) \cos 125^\circ \\ &= -200.178 \text{ m} \end{aligned}$$

$$\begin{aligned} \Delta y_4 &= (349 \text{ m}) \sin 125^\circ \\ &= 285.884 \text{ m} \end{aligned}$$

$$\begin{aligned} \Delta x_{tot} &= 119 \text{ m} - 16.8291 \text{ m} - 200.178 \text{ m} \\ &= -98.0072 \text{ m} \end{aligned}$$

$$\begin{aligned} \Delta y_{tot} &= -336 \text{ m} - 160.118 \text{ m} + 285.884 \text{ m} \\ &= -210.234 \text{ m} \end{aligned}$$

and

$$\begin{aligned} d &= \sqrt{(-98.0072 \text{ m})^2 + (-210.234 \text{ m})^2} \\ &= 231.956 \text{ m} \end{aligned}$$

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002 (part 2 of 2) 10.0 points

What is the direction (measured from due west, with counterclockwise positive) of the person's resultant displacement?

Correct answer: 65.0059°.

Explanation:

$$\begin{aligned} \tan\theta &= \frac{\Delta y_{tot}}{\Delta x_{tot}} \\ \theta &= \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) \\ &= \tan^{-1}\left(\frac{-210.234 \text{ m}}{-98.0072 \text{ m}}\right) \\ &= 65.0059^\circ \end{aligned}$$

south of west.

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003 10.0 points

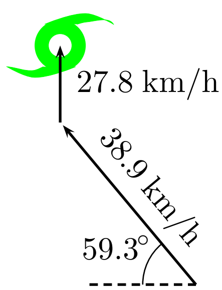
The eye of a hurricane passes over Grand Bahama Island. It is moving in a direction 59.3° north of west with a speed of 38.9 km/h. Exactly 2.89 hours later, the course of the hurricane shifts due north, and its speed slows to 27.8 km/h, as shown.

How far from Grand Bahama is the hurricane 4.42 h after it passes over the island?

Correct answer: 150.568 km.

Explanation:

$$\begin{aligned}\text{Let :}\quad \theta &= 59.3^\circ \text{ north of west}\,,\\ &= 120.7^\circ \text{ north of east}\,,\\ v_1 &= 38.9 \text{ km/h}\,,\\ \Delta t_1 &= 2.89 \text{ h}\,,\\ v_2 &= 27.8 \text{ km/h}\,, \quad \text{and}\\ \Delta t &= 4.42 \text{ h}\,.\end{aligned}$$



Since the second motion is directed due north,

$$\begin{aligned}\Delta x_{tot} &= v_{x,1}\Delta t_1\\ &= (v_1 \cos\theta)\,\Delta t_1\\ &= (38.9 \text{ km/h})\,(\cos 120.7^\circ)\,(2.89 \text{ h})\\ &= -57.3957 \text{ km}\,.\end{aligned}$$

$$\Delta t_2 = \Delta t - \Delta t_1$$

$$\begin{aligned}\Delta y_{tot} &= v_{y,1}\Delta t_1 + v_{y,2}\Delta t_2\\ &= (v_1 \sin\theta)\,\Delta t_1 + v_2\,(\Delta t - \Delta t_1)\\ &= (38.9 \text{ km/h})(\sin 120.7^\circ)(2.89 \text{ h})\\ &\quad + (27.8 \text{ km/h})(4.42 \text{ h} - 2.89 \text{ h})\\ & 139.199 \text{ km}\end{aligned}$$

Thus

$$\begin{aligned}d &= \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2}\\ &= \sqrt{(-57.3957 \text{ km})^2 + (139.199 \text{ km})^2}\\ &= \boxed{150.568 \text{ km}}\,.\end{aligned}$$

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004 10.0 points

Vector $\vec{A}$ has a magnitude of 3.9 units and vector $\vec{B}$ has a magnitude of 14 units. The two vectors make an angle of 79° with each other.

Find $\vec{A}\cdot\vec{B}$.

Correct answer: 10.4181 units^2.

Explanation:

$$\begin{aligned}\vec{A}\cdot\vec{B} &= \|\vec{A}\|\,\|\vec{B}\|\,(\cos\theta_{AB})\\ &= (3.9 \text{ units})\,(14 \text{ units})\,(\cos 79^\circ)\\ &= 10.4181 \text{ units}^2\end{aligned}$$

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005 (part 1 of 2) 10.0 points

Given: Two vectors

$$\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath}$$

and

$$\vec{B} = B_x\,\hat{\imath} + B_y\,\hat{\jmath}\,,$$

where $A_x = -1$, $A_y = 3$, $B_x = 3$, and $B_y = 2$.

Find the z component of $\vec{A}\times\vec{B}$.

Correct answer: -11.

Explanation:

It follows from the definition of cross product that

$$\begin{aligned}(\vec{A}\times\vec{B})_x &= A_y\,B_z - A_z\,B_y = 0\\ (\vec{A}\times\vec{B})_y &= A_z\,B_x - A_x\,B_z = 0\\ (\vec{A}\times\vec{B})_z &= A_x\,B_y - A_y\,B_x\\ &= (-3)\,(3) - (4)(2)\\ &= -11\,.\end{aligned}$$

006 (part 2 of 2) 10.0 points

Find the angle between $\vec{A}$ and $\vec{B}$.

Correct answer: 74.7449°.

Explanation:

$$\begin{aligned}\vec{A}\cdot\vec{B} &= A_x\,B_x + A_y\,B_y\\ &= (-1)\,(3) + (3)\,(2)\\ &= 3\,,\end{aligned}$$

$$\begin{aligned}A &= \sqrt{A_x^2 + A_y^2}\\ &= \sqrt{(-1)^2 + (3)^2}\\ &= 3.16228\,,\end{aligned}$$

and

$$\begin{aligned}B &= \sqrt{B_x^2 + B_y^2}\\ &= \sqrt{(3)^2 + (2)^2}\\ &= 3.60555\,.\end{aligned}$$

Therefore, using

$$\vec{A}\cdot\vec{B} = |A|\,|B|\cos\theta\,,$$

and solving for θ, gives

$$\begin{aligned}\theta &= \cos^{-1}\left[\frac{\vec{A}\cdot\vec{B}}{A\,B}\right]\\ &= \cos^{-1}\left[\frac{(3)}{(3.16228)\,(3.60555)}\right]\\ &= 1.30454 \text{ rad}\\ &= 74.7449^\circ\,.\end{aligned}$$

007 (part 1 of 2) 10.0 points

The vectors $\vec{A}$ and $\vec{B}$ are given by

$$\begin{aligned}\vec{A} &= 1.83\,\hat{\imath} + 6.63\,\hat{\jmath}\\ \vec{B} &= -1.96\,\hat{\imath} + 2.35\,\hat{\jmath}\end{aligned}$$

Find the scalar product $\vec{A}\cdot\vec{B}$.

Correct answer: 11.9937.

Explanation:

$\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = 1$ and $\hat{\imath}\cdot\hat{\jmath} = 0$, so

$$\begin{aligned}\vec{A}\cdot\vec{B} &= A_x\,B_x + A_y\,B_y\\ &= (1.83)\,(-1.96) + (6.63)\,(2.35)\\ &= \boxed{11.9937}\,.\end{aligned}$$

008 (part 2 of 2) 10.0 points

Find the angle between $\vec{A}$ and $\vec{B}$.

Correct answer: 55.26°.

Explanation:

The magnitudes of $\vec{A}$ and $\vec{B}$ are given by

$$\begin{aligned}A &= \sqrt{A_x^2 + A_y^2} = \sqrt{1.83^2 + 6.63^2} = 6.87792\\ B &= \sqrt{B_x^2 + B_y^2} = \sqrt{-1.96^2 + 2.35^2} = 3.06008\,.\end{aligned}$$

$$\begin{aligned}\vec{A}\cdot\vec{B} &= A\,B\cos\theta\\ \cos\theta &= \frac{\vec{A}\cdot\vec{B}}{A\,B}\\ &= \arccos\frac{\vec{A}\cdot\vec{B}}{A\,B}\\ &= \arccos\frac{11.9937}{(6.87792)\,(3.06008)}\\ &= \arccos(0.569853)\\ &= 55.26^\circ\,.\end{aligned}$$