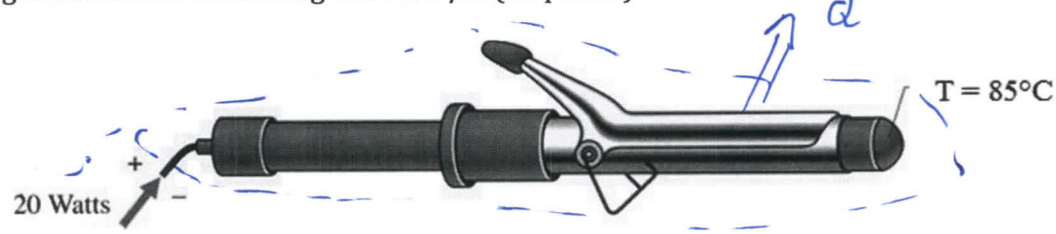
MAE 341-03 Final Exam **May 1, 2013** **Name:** ____________________

Problems (Open Book)

1. A 20 W curling iron has a surface temperature of 85°C. Determine the rate of entropy generation for the curling iron in W/K. (10 points)



1st Law

$$0 = \dot{Q} - \dot{W} \qquad \dot{W} = -20\ \text{W}$$

$$\dot{Q} = -20\ \text{W} \qquad \frac{dE}{dt} = 0$$

2nd Law

$$0 = \frac{\dot{Q}}{T_b} + \dot{\sigma} \qquad \frac{dS}{dt} = 0$$

$$\dot{\sigma} = -\frac{\dot{Q}}{T_b} = \frac{20\ \text{W}}{358\ \text{K}} = 0.0559\ \frac{\text{W}}{\text{K}}$$

$0 = -\dot{Q}_{out} + Q_{in} + W$

2. A heat pump operating at steady-state maintains the inside of a building at 20°C while transferring heat from a well at 10°C. The heat pump transfers 120,000 kJ/h to the building. Over 14 days the heat pump consumes 1490 kW·h of electricity.
   (a) Find the amount of heat in kJ transferred from the well by the heat pump during the same 14 day period. (5 points)
   (b) What is the coefficient of performance for the heat pump? (5 points)
   (c) What is the maximum coefficient of performance for this heat pump when operating between the well (10°C) and the building (20°C)? (5 points)

(a) $W_{cycle} = Q_{out} - Q_{in}$

$Q_{out} = 120{,}000 \frac{kJ}{h} \left(\frac{24h}{day}\right) 14\ days$

$= 40.32 \times 10^6\ kJ$

$W_{cycle} = 1490\ kW \cdot h \left(\frac{1\ kJ/s}{1\ kW}\right) \cdot \frac{3600\ s}{1\ h} = 5.364 \times 10^6\ kJ$

$Q_{in} = Q_{out} - W_{cycle}$

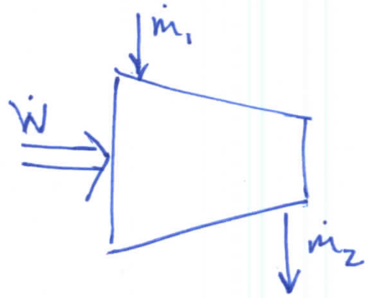
$= (40.3\cancel{6}2 - 5.364) \times 10^6\ kJ$

$= 34.956 \times 10^6\ kJ$

(b) $\beta_{HP} = \frac{Q_{out}}{W_{cycle}} = 7.52$

(c) $\beta_{HP,max} = \frac{T_H}{T_H - T_C} = \frac{293\ K}{(293 - 283)\ K} = 29.3$

3. Nitrogen (N_2) at 1 bar, 37°C is compressed to 10 bar using a well-insulated compressor. The mass flow rate is 1000 kg/h and the specific heat ratio of nitrogen is $k = 1.391$.
   (a) What is the minimum theoretical work needed to compress the gas? (10 points)
   (b) If the exit temperature of the compressor is 397°C, what is the isentropic efficiency of the compressor? (10 points)
   (c) Suppose the compressor is poorly insulated, but the exit temperature is still 397°C. Would the isentropic efficiency be higher or lower? Justify your answer using the 1st and 2nd Laws. (5 points)



(a) $\dot{m}_1 = \dot{m}_2$ (1 in, 1 out)

$$0 = \cancel{\dot{Q}} - \dot{W} + \dot{m}(h_1 - h_2)$$

$$\dot{W} = \dot{m} c_p (T_1 - T_2)$$

① Well-ins.
② Steady-state
③ $\Delta PE + \Delta KE \sim 0$
④ Ideal gas, $c_p = \text{const.}$

$$c_p = \frac{kR}{k-1} \qquad R_{N_2} = \frac{8.314 \frac{J}{mol\,K}}{28 \frac{g}{mol}} = 0.2969 \frac{kJ}{kg \cdot K}$$

$$= \frac{1.391(0.297)}{0.391} \frac{kJ}{kg \cdot K}$$

$$= 1.056 \frac{kJ}{kg \cdot K}$$

$$\dot{W}_s = \dot{m} c_p (T_1 - T_{2s}) \qquad T_{2s} = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 592.2\,K$$

$$= 310\,K (10)^{\frac{0.391}{1.391}}$$

$$\dot{W}_s = 1000 \frac{kg}{h} \left(\frac{1h}{3600s}\right)\left(1.056 \frac{kJ}{kg \cdot K}\right)(310 - 592.2)K$$

$$\dot{W}_s = -82.78\ kW$$

(b) $\dot{W} = \dot{m} c_p (T_1 - T_2)$

$$= 1000 \frac{kg}{h} \left(\frac{1h}{3600 s}\right)\left(1.056 \frac{kJ}{kg \cdot K}\right)(310 - 670) K$$

$$= -105.6 \text{ kW}$$

$$\eta_c = \frac{\dot{W}_s}{\dot{W}} = 78.4\%$$

(c) If there is $\dot{Q}$ ~~out~~ then

<u>1st Law</u> $\cancel{0 = \dot{W}}$ $0 = \dot{Q} - \dot{W} + \dot{m} c_p (T_1 - T_2)$

<u>2nd Law</u> $0 = \frac{\dot{Q}}{T_b} + \dot{m}\left[c_p \ln\left(\frac{T_1}{T_2}\right) - R \ln\left(\frac{P_1}{P_2}\right)\right] + \dot{\sigma}$

$$0 = \frac{\dot{Q}}{T_b} + \dot{m} R\left[\frac{k}{k-1}\left(\frac{T_1}{T_2}\right) - \ln\left(\frac{P_1}{P_2}\right)\right] + \dot{\sigma}$$

$$-\dot{\sigma} = \frac{\dot{Q}}{T_b} + \left(1000 \frac{kg}{h}\right)\left(0.297 \frac{kJ}{kg\,K}\right)\left(\frac{1h}{3600 s}\right) \times \left[\frac{1.391}{0.391} \ln\left(\frac{310}{670}\right) - \ln(0.1)\right]$$

$$\dot{\sigma} = -\frac{\dot{Q}}{T_b} + 0.0825\left[-0.43923\right] \frac{kW}{K}$$

$\dot{\sigma} \geq 0$ + for non-ideal $\dot{\sigma} > 0$

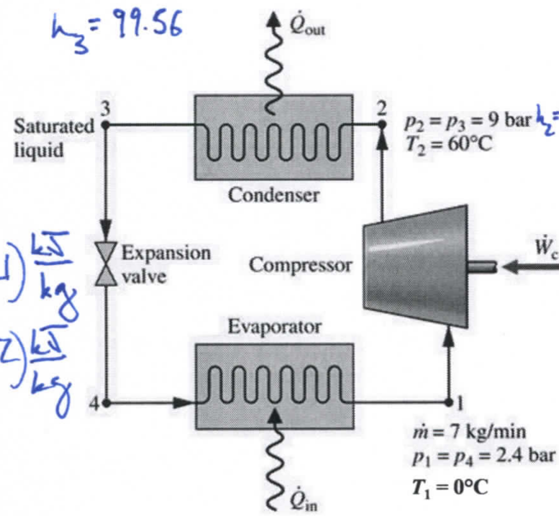
So $\dot{Q} < 0$ (out) + $\cancel{|\dot{Q}| >}$ $\frac{\dot{Q}}{T_b} < -0.0362 \frac{kW}{K}$

$\Rightarrow$ $\cancel{\dot{W} < -105.6}$ $\dot{W}_{in}$ must be larger,

so η_c will be smaller.

4. A heat pump using R-134a operates at steady-state as shown below.
   (a) What is the work input into the compressor? (10 points)
   (b) What is the coefficient of performance for this heat pump? (10 points)
   (c) An inventor claims to have a turbine that can replace the expansion valve and recover 50% of the power required for the compressor. Is this a valid claim? Justify your answer with a number. (5 points)

$h_3 = 99.56$



$h_2 = 293.21 \frac{kJ}{kg}$

$h_1 = 248.89 \frac{kJ}{kg}$

(a) $\dot{W}_c = \dot{m}(h_1 - h_2)$

$= 7 \frac{kg}{min} \frac{1\,min}{60\,s} \times (248.89 - 293.21) \frac{kJ}{kg}$

$= 0.1167 \frac{kg}{s} (-44.32) \frac{kJ}{kg}$

$= -5.17$ kW

(b) $\beta_{HP} = \frac{\dot{Q}_{out}}{\dot{W}_c} \cancel{= \frac{\dot{m}(h_2 - h_3)}{h_1 - h_2}} = \frac{22.6\,kW}{5.17\,kW} = 4.37$

$\dot{Q}_{out} = \dot{m}(h_3 - h_2)$

$= -22.60$ kW

(c) In a best case for the turbine

$s_{4s} = s_3 = 0.3656 \frac{kJ}{kg \cdot K}$ Sat. ~~vap~~ mixture at p_1

$x_{4s} = \frac{s_{4s} - s_f}{s_g - s_f} = 0.259 \Rightarrow h_{4s} = 95.05 \frac{kJ}{kg}$

$\dot{W}_t = 0.1167 \frac{kg}{s} (99.56 - 95.05) \frac{kJ}{kg}$

$= 0.53\,kW < 0.5 \cdot 5.17 \Rightarrow$ Not possible