Correcting for pixel distances

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1 Problem

We wish for the "depth image" to be constant on a plane. That is, we want the gradient of the "depth image" to be zero. "Depth image" is in quotes because the actual depth image produced by the scanner does not have this property.

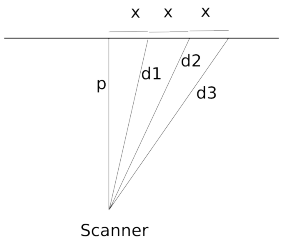

Figure 1: Uniformly spaced point on the object

'p' in Figure 1 is the perpendicular distance from the scanner to the plane. We want the differences of all neighboring "depths" to be equal, that is:

$$d_1 - p = d_2 - d_1 = d_3 - d_2 \tag{1}$$

From trigonometry, we compute:

$$d_1 = \frac{x}{sin(atan(\frac{x}{p}))} \tag{2}$$

likewise,

$$d_2 = \frac{2x}{sin(atan(\frac{2x}{p}))} \tag{3}$$

Choosing $x = 2$ and $p = 5$, we have:

- $d_1 = 5.38$

- $d_2 = 6.40$
- $d_3 = 7.81$

The differences are therefore

- $d_1 - p = 5.38 - 5 = 0.38$
- $d_2 - d_1 = 6.40 - 5.38 = 1.02$
- $d_3 - d_2 = 7.81 - 6.40 = 1.41$

These distances are not constant, as we would expect. This is because the pixel width is not constant. The pixel width is related to the angle.

The angle made by the first ray is 0. The angle made by the ray named d_1 is .38. The angle made by d_2 is .674. Thus, for a distance of x to $2x$ the angle does not double, it increases by a factor of only $\frac{.674}{.38} = 1.77$. Therefore, the angle between d_1 and d_2 is $.674 - .38 = .29$. Likewise the next angle is .876, so the angle between d_2 and d_3 is $.876 - .674 = .202$.

Thus, if we normalize the distances by their pixel width, we get:

- $(d_1 - p)' = \frac{d_1 - p}{w_1} = \frac{.38}{.38} = 1$
- $(d_2 - d_1)' = \frac{d_2 - d_1}{w_2} = \frac{1.02}{.29} = 3.51$
- $(d_3 - d_2)' = \frac{d_3 - d_2}{w_3} = \frac{1.41}{.202} = 6.98$