Correcting for pixel distances

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1 Problem

We wish for the "depth image" to be constant on a plane. That is, we want the gradient of the "depth image" to be zero. "Depth image" is in quotes because the actual depth image produced by the scanner does not have this property.

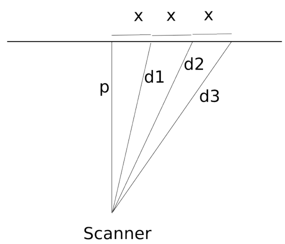

Figure 1: Uniformly spaced point on the object

'p' in Figure 1 is the perpendicular distance from the scanner to the plane. We want the differences of all neighboring "depths" to be equal, that is:

$$d_1 - p = d_2 - d_1 = d_3 - d_2 \tag{1}$$

From trigonometry, we compute:

$$d_1 = \frac{x}{sin(atan(\frac{x}{p}))} \tag{2}$$

likewise,

$$d_2 = \frac{2x}{sin(atan(\frac{2x}{p}))} \tag{3}$$

Choosing $x = 2$ and $p = 5$, we have:

- $d_1 = 5.38$

- $d_2 = 6.40$
- $d_3 = 7.81$

The differences are therefore

- $d_1 - p = 5.38 - 5 = 0.38$
- $d_2 - d_1 = 6.40 - 5.38 = 1.02$
- $d_3 - d_2 = 7.81 - 6.40 = 1.41$

These distances are not constant, as we would expect.

We must introduce a difference function which does produce a constant for this case. This can be constructed from the law of cosines:

$$diff(d_n, d_{n+1}) = \sqrt{d_n^2 + d_{n+1}^2 - 2d_n d_{n+1} cos(\Theta(d_n, d_{n+1}))} \tag{4}$$

where $\Theta(d_n, d_{n+1})$ is the angle between ray n and ray $n+1$.

The angles between the lines are:

- $\Theta(p, d_1) = .38$
- $\Theta(d_1, d_2) = .29$
- $\Theta(d_2, d_3) = .202$

Therefore the new distance function values are:

- $f(p, d_1) = \sqrt{p^2 + d_1^2 - 2 * p * d_1 cos(Theta(p, d_1))} = |\sqrt{5^2 + 5.38^2 - 5 * 5.38 * cos(.38)} = 1.99$
- $f(d_1, d_2) = \sqrt{d_1^2 + d_2^2 - 2 * d_1 * d_2 cos(Theta(d_1, d_2)} = \sqrt{5.38^2 + 6.40^2 - 2 * 5.38 * 6.40 * cos(.29)} = 1.9789$
- $f(d_2, d_3) = \sqrt{d_2^2 + d_3^2 - 2 * d_2 * d_3 cos(Theta(d_2, d_3)} = \sqrt{6.40^2 + 7.81^2 - 2 * 6.40 * 7.81 * cos(.202)} = 2.0052$

Which are all the same!