Looking at your last line I would disagree with one thing.
For negative m:

$$\frac{\partial}{\partial\phi}Y_l^{-m}=\sqrt{\frac{2\,l+1}{4\,\pi}\;\frac{(l+m)}{(l-m)\,!}}\;e^{-im\theta}\left[-m\cot(\phi)\,P_l^{-m}(\cos\phi)+P_l^{-m+1}(\cos\phi)\right]$$

That is I changed the argument of Exp from imθ to -imθ
From that we can distribute, I also split the second θ function:

$$\frac{\partial}{\partial\phi}Y_l^{-m}=\sqrt{\frac{2\,l+1}{4\,\pi}\;\frac{(l+m)}{(l-m)\,!}}\;\left[-m\cot(\phi)\,e^{-im\theta}\,P_l^{-m}(\cos\phi)+e^{-i\theta}\,e^{-i\,(m-1)\,\theta}\,P_l^{-m+1}(\cos\phi)\right]$$

This becomes:

$$\sqrt{\frac{2\,l+1}{4\,\pi}\;\frac{(l+m)}{(l-m)\,!}}\;\left[-m\cot(\phi)\,Y_l^{-m}+e^{-i\phi}\,Y_l^{-m+1}\right]$$