1. **The Line Element**

In cylindrical coordinates t, z, r, ϕ,

$$ds^2 = dt^2\,\left(L^2-1\right) + 2\;r\,L\;d\phi\,dt + r^2\,d\phi^2 + dz^2 + dr^2$$

where $0 < L < 1$ is a constant.

The proper acceleration of the vector

$$U_\mu = \sqrt{1-L^2}\;\vec{\partial_t} - rL\vec{\partial_\phi}/\sqrt{1-L^2}$$

is $\dot{U}_\nu = (\nabla_\mu U_\nu)U^\mu$, the covariant derivative of U_ν projected in the direction U^μ. The covector $g^{\mu\nu}U_\nu$ has only a time (first) component so the only terms of $D_{\mu\nu} = \nabla_\mu U_\nu$ which can be selected by the contraction of $D_{\mu\nu}$ with U^ν are in the first column, $D_{1,k}$.

Using $\nabla_\mu U_\nu = \partial_\mu U_\nu - \Gamma_{\mu\nu}{}^\alpha U_\alpha$ there are eight instances where $\Gamma_{\mu\nu}{}^\alpha U_\alpha$ is non-zero listed below in the 'Trace output section' in the pdf.

The first two lines are equivalent to

$$\begin{aligned} D_{12} &= -\Gamma_{12}{}^1 U_1 - \Gamma_{12}{}^4 U_4 \\ &= \frac{L\,\left(L^3-L\right)}{2\,r\,\sqrt{1-L^2}} - \frac{\left(L-1\right)\,L^2\,\left(L+1\right)}{2\,r\,\sqrt{1-L^2}} \\ &= 0 \end{aligned}$$

and since there are no other candidates in the first column of $D_{\mu\nu}$ this is sufficient to eliminate any proper acceleration.

2. **Trace output**

The amended script reports this as it loops through μ, ν, α. These are the first three digits.

$$121\;\; dU = 0\;\; U(1) = -\frac{\left(L-1\right)\,\left(L+1\right)}{\sqrt{1-L^2}}\;\; mcs = \frac{L^2}{2\,r} >> -\frac{\left(L-1\right)\,L^2\,\left(L+1\right)}{2\,r\,\sqrt{1-L^2}}$$

$$124\;\; dU = 0\;\; U(4) = -\frac{r\,L}{\sqrt{1-L^2}}\;\; mcs = -\frac{L^3-L}{2\,r^2} >> \frac{L\,\left(L^3-L\right)}{2\,r\,\sqrt{1-L^2}}$$

$$211\;\; dU = 0\;\; U(1) = -\frac{\left(L-1\right)\,\left(L+1\right)}{\sqrt{1-L^2}}\;\; mcs = \frac{L^2}{2\,r} >> -\frac{\left(L-1\right)\,L^2\,\left(L+1\right)}{2\,r\,\sqrt{1-L^2}}$$

$$214\;\; dU = 0\;\; U(4) = -\frac{r\,L}{\sqrt{1-L^2}}\;\; mcs = -\frac{L^3-L}{2\,r^2} >> \frac{L\,\left(L^3-L\right)}{2\,r\,\sqrt{1-L^2}}$$

$$241\;\; dU = 0\;\; U(1) = -\frac{\left(L-1\right)\,\left(L+1\right)}{\sqrt{1-L^2}}\;\; mcs = \frac{L}{2} >> -\frac{\left(L-1\right)\,L\,\left(L+1\right)}{2\,\sqrt{1-L^2}}$$

$$244\;\; dU = 0\;\; U(4) = -\frac{r\,L}{\sqrt{1-L^2}}\;\; mcs = -\frac{L^2-2}{2\,r} >> \frac{L\,\left(L^2-2\right)}{2\,\sqrt{1-L^2}}$$

$$421\;\; dU = -\frac{L}{\sqrt{1-L^2}}\;\; U(1) = -\frac{\left(L-1\right)\,\left(L+1\right)}{\sqrt{1-L^2}}\;\; mcs = \frac{L}{2} >> -\frac{\left(L-1\right)\,L\,\left(L+1\right)}{2\,\sqrt{1-L^2}}$$

$$424\;\; dU = -\frac{L}{\sqrt{1-L^2}}\;\; U(4) = -\frac{r\,L}{\sqrt{1-L^2}}\;\; mcs = -\frac{L^2-2}{2\,r} >> \frac{L\,\left(L^2-2\right)}{2\,\sqrt{1-L^2}}$$

References

[1] Leslie E. Ballentine
Quantum Mechanics - A Modern Development
World Scientific (1998)