$$\sum_{n=1}^{\infty}\frac{n!x^n}{5*11*17\cdots(6n-1)}$$

Ratio Test

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\dfrac{(n+1)!x^{n+1}}{5*11*17\cdots(6n-1)(6n+1)}}{\dfrac{n!x^n}{5*11*17\cdots(6n-1)}}\right|$$

$$= \left|\frac{(n+1)!x^{n+1}}{5*11*17\cdots(6n-1)(6n+1)} * \frac{5*11*17\cdots(6n-1)}{n!x^n}\right|$$

$$= \left|\frac{(n+1)x}{(6n+1)} * \frac{1}{1}\right| = \lim_{n\to\infty}|x|\frac{(n+1)}{(6n+1)} = \lim_{n\to\infty}|x| * \frac{1}{6}$$

$$|x| * \frac{1}{6} < 1 \Leftrightarrow |x| < 6 \Leftrightarrow -6 < x < 6$$

When x = 6,-6 (take absolute value)

$$\sum_{n=1}^{\infty}\frac{n!x^n}{5*11*17\cdots(6n-1)} = \sum_{n=1}^{\infty}\frac{n!6^n}{5*11*17\cdots(6n-1)}$$

$$= \left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\dfrac{(n+1)!6^{n+1}}{5*11*17\cdots(6n-1)(6n+1)}}{\dfrac{n!6^n}{5*11*17\cdots(6n-1)}}\right|$$

$$= \left|\frac{(n+1)!6^{n+1}}{5*11*17\cdots(6n-1)(6n+1)} * \frac{5*11*17\cdots(6n-1)}{n!6^n}\right|$$

$$= \left|\frac{(n+1)*6}{(6n+1)}\right| = \lim_{n\to\infty}\left|\frac{6n+6}{6n+1}\right| = \lim_{n\to\infty}1$$