height of the waves), that surface tension effects are not important, and that water is incompressible.

c) Find the group velocity of the wavefront and consider two limiting cases $\lambda \gg d$, $\lambda \ll d$.

## 1.48 Suspension Bridge (Stony Brook)

A flexible massless cable in a suspension bridge is subject to uniform loading along the $x$-axis. The weight of the load per unit length of the cable is $w$, and the tension in the cable at the center of the bridge (at $x = 0$) is $T_0$ (see Figure P. 1.48).

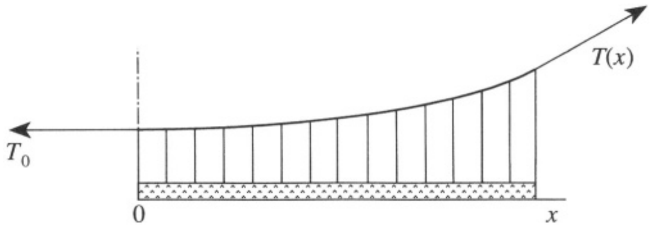


Figure **P.1.48**

a) Find the shape of the cable at equilibrium.
b) What is the tension $T(x)$ in the cable at position $x$ at equilibrium?

## 1.49 Catenary (Stony Brook, MIT)

A flexible cord of uniform density $\rho$ and fixed length $l$ is suspended from two points of equal height (see Figure P. 1.49). The gravitational acceleration is taken to be a constant $g$ in the negative $z$ direction.

a) Write the expressions for the potential energy $U$ and the length $l$ for a given curve $z = z(x)$.
b) Formulate the Euler-Lagrange equations for the curve with minimal potential energy, subject to the condition of fixed length.
c) Show that the solution of the previous equation is given by $z = A\cosh(x/A) + B$, where $A$ and $B$ are constants. Calculate $U$ and $l$ for this solution.

c) The group velocity of the waves is

$$u = \frac{\partial \omega}{\partial k} = \frac{\sqrt{g}}{2\sqrt{k \tanh kd}} \left( \tanh kd + \frac{kd}{\cosh^2 kd} \right) \qquad \text{(S.1.47.13)}$$

Consider two limiting cases:

1) $kd \gg 1$, $d \gg \lambda$—short wavelength waves. Then

$$u \approx \frac{1}{2}\sqrt{g/k} = \frac{1}{2}\sqrt{g\lambda/2\pi}$$

2) $kd \ll 1$, $d \ll \lambda$—long wavelength waves. Then $u \approx \sqrt{gd}$.

## 1.48 Suspension Bridge (Stony Brook)

a) We use an elementary method to solve this problem. The conditions for a static equilibrium are

$$T(x+dx) \cdot \cos\theta(x+dx) - T(x)\cos\theta(x) = F_x = 0 \qquad \text{(S.1.48.1)}$$

$$T(x+dx) \cdot \sin\theta(x+dx) - T(x)\sin\theta(x) = w\,dx \qquad \text{(S.1.48.2)}$$

(see Figure S.1.48). (S.1.48.1) and (S.1.48.2) can be rewritten in the form

$$\frac{d}{dx}\left[T(x)\cos\theta(x)\right] = 0 \qquad \text{(S.1.48.3)}$$

$$\frac{d}{dx}\left[T(x)\sin\theta(x)\right] = w\,dx \qquad \text{(S.1.48.4)}$$

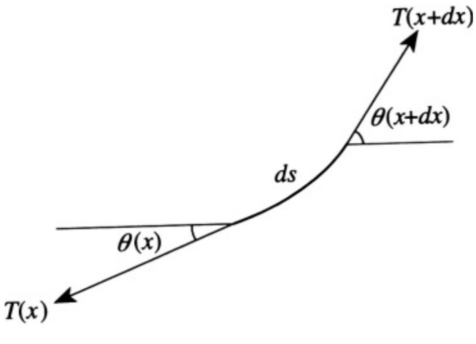


Figure S.1.48

Integrating (S.1.48.3) and (S.1.48.4), we obtain

$$T(x)\sin\theta(x) = C_0 + wx \tag{S.1.48.5}$$

$$T(x)\cos\theta(x) = C_1 \tag{S.1.48.6}$$

At $x = 0,\ \theta = 0$, so $C_0 = 0$; $C_1 = T_0$ and dividing (S.1.48.5) by (S.1.48.6), we have

$$\tan\theta(x) = \frac{dy}{dx} = \frac{w}{T_0}x \tag{S.1.48.7}$$

From (S.1.48.7) we find the shape of the suspension bridge, which is parabolic

$$y = y_0 + \frac{wx^2}{2T_0} \tag{S.1.48.8}$$

b) To find the tension $T(x)$ at $x \neq 0$ ($\theta \neq 0$), multiply (S.1.48.5) by (S.1.48.6).

$$T^2(x)\sin\theta\cos\theta = T_0wx \tag{S.1.48.9}$$

$$T^2(x)\frac{\tan\theta}{1+\tan^2\theta} = T_0wx \tag{S.1.48.10}$$

$$T^2(x) = T_0wx\left(\tan\theta + \frac{1}{\tan\theta}\right) = T_0wx\left(\frac{w}{T_0}x + \frac{T_0}{wx}\right) = T_0^2 + w^2x^2$$

So

$$T(x) = T_0\sqrt{1+(wx/T_0)^2} \tag{S.1.48.11}$$

## 1.49 Catenary (Stony Brook, MIT)

a) Write the expressions for the length $l$ and potential energy $U$ (see Figure S.1.49) using

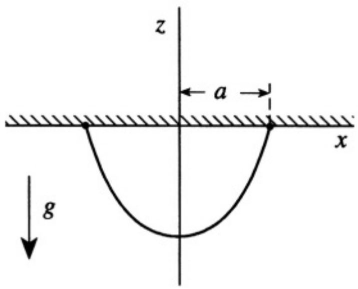


Figure S.1.49