Symmetry Factors of Feynman Diagrams for Scalar Fields

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Abstract

We calculate the symmetry factors of diagrams for real and complex scalar fields in general form using an analysis of the Wick expansion for Greens functions. We separate two classes of symmetry factors: factors corresponding to connected diagrams and factors corresponding to vacuum diagrams. The symmetry factors of vacuum diagrams play an important role in constructing the effective action and phase transitions in cosmology. In the complex scalar field theory, diagrams with different topologies can contribute the same, and the inverse symmetry factor for the total contribution is therefore the sum of the inverse symmetry factors, i.e., $1/S = \sum_i (1/S_i)$.

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1 Introduction

In quantum field theory, physical processes are described by the elements of the S-matrix, which are in turn given by Feynman diagrams. One important task in calculating these diagrams is determining their symmetry factors (see, e.g., [1]). Fortunately,

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there are now various convenient computer programs (for instance, FeynArts [2] or QGRAF [3]) for constructing Feynman diagrams in different field theories. We note that QGRAF does not work with vacuum diagrams, which play an important role in effective theories. In a series of papers, Kastening and coauthors [4] developed an alternative systematic approach for constructing all Feynman diagrams based on considering a Feynman diagram as a functional of its graphical elements. We stress that only real fields were considered in all these papers, and complex fields were outside the scop

Our aim here is to derive a general formula describing the case of complex scalar fields (it, of course, would also hold in the case of real fields). This formula turns out to be easily understood and is therefore very useful for those physicists who have not developed good skills in computer disciplines. Below, we show that the case of complex fields has very special features that are absent in the case of real fields.

We return to our questions. What is the symmetry factor? How is it constructed? We consider a pth-order expansion of the n-point correlation function in a real scalar theory with the interaction $\mathcal{L}_{int} = (\lambda/4!)\phi^4$:

$$(1/p!)(1/4!)^p \langle 0|T[\phi(x_1)\phi(x_2)...\phi(x_n)\phi^4(y_1)\phi^4(y_2)...\phi^4(y_p)]|0\rangle, \tag{1}$$

where the factor $(i\lambda)^p$ and integrations over $y_1,\ y_2, ..., y_p$ are omitted because they are always presumed in the Feynman rules. Our task is to count the number of different contractions giving the same expression (corresponding to a Feynman diagram) [5]. This number is equal to N/D, the number of all possible contractions divided by the number of identical contractions. The overall constant of the diagram then becomes $S^{-1} \equiv (1/p!)(1/4!)^p N/D$. The number S, called the symmetry factor of the diagram, generally differs from unity. Further, the numerator N is a product of p! interchanges of the vertices $y_1,\ y_2, ...,\ y_p$ and N_i self-contractions of the vertex y_i $(i = 1, 2, ..., p)$ and placements of contractions into this vertex. The value of N_i is 4! if there is no self-contraction, 4!/2 if there is one self-contraction (single bubble), and 4!/8 if there are two self-contractions (double bubble). Hence, $N = p! \prod_i N_i = [p!(4!)^p]/[2^s 8^d]$, where s and d are the respective numbers of single-bubble and double-bubble vertices. Because a double bubble contains two single bubbles, the total number of single bubbles is $\beta = s + 2d$. We can rewrite $N = [p!(4!)^p]/[2^\beta 2^d]$.

In contrast, determining the denominator D is not so easy. Briefly, we evaluate it as follows. First, we consider the interchange of vertexvertex contractions. If there are n contractions, then we have n! interchanges. Second, we consider the interchange of the vertices $y_1, ..., y_p$ giving identical contractions, i.e., an identical set of Feynman propagators. In this case, there are $d!$ interchanges of d-type vertices times g' interchanges of the remaining vertices. The result is $D = g'd! \prod_{n=2,3\ldots}(n!)^{\alpha_n}$, where α_n is the number of vertex pairs with n contractions. The symmetry factor is given by

$$S = g'd! \prod_n (n!)^{\alpha_n} 2^d 2^\beta. \tag{2}$$

Determining g' is nontrivial [6] and sometimes leads to significant problems. In the literature, only the symmetry factors in the real scalar field theory with connected diagrams are presented; those for the vacuum diagrams and also those for the complex fields are absent. We note that vacuum diagrams have applications in particle physics and cosmology, such as in the effective action and phase transition (see, e.g., [1], [5] and also [7] for a recent implication). A formula for calculating the symmetry factors of such diagrams is needed; our aim here is to explicitly derive such a formula for the symmetry factor in the real scalar theory. We do this by applying Wick's theorem and in the process show that the vacuum diagrams are factored [8] explicitly order by perturbation theory order. We also study its generalizations to complex scalar fields. We also list symmetry factors corresponding to Feynman diagrams in both theories.

This work is organized as follows. In Sec. 2, we present some notation. In Sec. 3, we formulate the symmetry factor for the real scalar theory. In Sec. 4, we generalize the formula to complex fields and also consider the special features existing only in the complex theory. In Sec. 5, we summarize our results and draw conclusions. The appendix is devoted to Feynman diagrams and the corresponding symmetry factors in both theories.

2 Notation

We recall some ingredients of the S-matrix approach. The time-evolution operator is given in terms of the action as [9]

$$\begin{aligned} U(t_1, t_2) &= T \exp[iS_{int}(t_1, t_2, \hat{\varphi})] \\ &= N\left\{\exp\left(\frac{1}{2}\frac{\delta}{\delta\varphi}\Delta\frac{\delta}{\delta\varphi}\right)\exp[iS_{int}(t_1, t_2, \varphi)]\right\}|_{\ldots}, \end{aligned} \tag{3}$$

where symbol $|_{\ldots}$ indicates that after differentiation, the classical fields φ_i are replaced with the quantized ones $\hat{\varphi}_i$ and T and N denote the time-ordering and normal-ordering operators. The S-matrix is the limit of the time-evolution operator as $t_1 \to -\infty$ and $t_2 \to +\infty$. The c-number function $\Delta(x, x')$ (Feynman propagator) is defined as

$$\Delta(x, x') = T[\hat{\varphi}(x)\hat{\varphi}(x')] - N[\hat{\varphi}(x)\hat{\varphi}(x')]. \tag{4}$$

The formula [9]

$$T\left\{\prod_{i=1}^{n}F_i(\hat{\varphi})\right\}=N\left\{\exp\left[\frac{1}{2}\sum_i\frac{\delta}{\delta\varphi_i}\Delta\frac{\delta}{\delta\varphi_i}+\sum_{i<k}\frac{\delta}{\delta\varphi_i}\Delta\frac{\delta}{\delta\varphi_k}\right]\prod_{i=1}^{n}F_i(\varphi_i)\right\}|_{...} \quad (5)$$

is useful for our further presentation. We note that the first term in the right-hand side of (5) is present only in the real field theory.

We recall that every Feynman diagram, as mentioned in the introduction, has a symmetry factor. In [1, 10], it has the form given by

$$S=g2^{\beta}\prod_{n=2,3...}(n!)^{\alpha_n}, \quad (6)$$

where α_n is the number of pairs of vertices connected by n identical self-conjugate lines, β is the number of lines connecting a vertex to itself, and g is the number of permutations of vertices that leave the diagram unchanged with fixed external lines. We note that the factor 2^{β} comes from the factor $1/2$ in the first term in the r.h.s of (5). We also note that formula (6) works only for connected diagrams but not for vacuum diagrams. We derive the symmetry factor in the general case as sketched in (2) for the real ϕ^4 theory.

3 Symmetry factors in real scalar theory

We consider the model with the interaction Lagrangian

$$\mathcal{L}^r_{int}=\frac{\lambda}{4!}\phi^4. \quad (7)$$

It is well known that there is a direct connection between the S-matrix elements and the Green's functions defined by the expansion $G(x_1,x_2,...,x_n)=\sum_{p=0}^{\infty}G^{(p)}(x_1,x_2,...,x_n)$ where the pth-order term has the form

$$\begin{aligned} G^{(p)}(x_1,x_2,...,x_n) &= \frac{i^p}{p!}\int_{-\infty}^{\infty}d^4y_1...d^4y_p\langle 0|T[\phi(x_1)...\phi(x_n) \\ &\quad \mathcal{L}^r_{int}(\phi(y_1))...\mathcal{L}^r_{int}(\phi(y_p))]|0\rangle. \end{aligned} \quad (8)$$

This term is called the pth-order Green's function. The full Green's function $G(x_1,...,x_n)$ contains every n–point diagram in the theory, both connected and disconnected.

We recall that the four fields in $L^r_{int}(\phi(y))$ are taken at equal times. Applying (5) for Lagrangian (7), we obtain

$$\phi^4(y) \sim T[\phi^4(y)] = N[\phi^4(y)] + 6N[\phi^2(y)]\dot{\Delta} + 3\dot{\Delta}\dot{\Delta}, \tag{9}$$

where $\dot{\Delta} \equiv \Delta(y, y)$ denotes the bubble diagram $\bigcirc$. We let a, b and c denote the three terms in (9)

$$a \equiv N[\phi^4(y)], \quad b \equiv N[\phi^2(y)]\dot{\Delta}, \quad c \equiv \dot{\Delta}\dot{\Delta}. \tag{10}$$

Then we can rewrite (9) as

$$\phi^4 \sim T[\phi^4] = a + 6b + 3c. \tag{11}$$

Green's function (8) is invariant under permutations of the interaction Lagrangians. Hence, the product of these Lagrangians can be expanded as a sum of monomials in a, b and c such that all terms $a^p b^q c^t$ with given p, q and t are equivalent under integration. The overall coefficients of the monomials in the expansion can be extracted using the multinomial formula

$$\begin{aligned} (x_1 + x_2 + \cdots + x_r)^p &= \sum_{p_1, p_2, \ldots, p_r} \frac{p!}{p_1! p_2! \cdots p_r!} x_1^{p_1} \cdots x_r^{p_r}, \\ \text{with} \quad p_1 + p_2 + p_3 + \cdots + p_r &= p. \end{aligned} \tag{12}$$

Equation (8) then becomes

$$\begin{aligned} G^{(p)}(x_1, x_2, ..., x_n) &= \frac{1}{p!}\left(\frac{i\lambda}{4!}\right)^p \sum_{p_1+p_2+p_3=p} \frac{p!}{p_1!p_2!p_3!} \int_{-\infty}^{\infty} d^4y_1 ... d^4y_p \\ &\quad \times \langle 0|T[\phi(x_1)...\phi(x_n) a^{p_1}(6b)^{p_2}(3c)^{p_3}]|0\rangle, \end{aligned} \tag{13}$$

where the variables in the integrand have the clear meaning

$$a^{p_1}b^{p_2}c^{p_3} = a(y_1)a(y_2)...a(y_{p_1})b(y_{p_1+1})b(y_{p_1+2})...b(y_{p_1+p_2})c(y_{p_1+p_2+1})c(y_{p_1+p_2+2})...c(y_p).$$

For the further presentation, we omit the summations and integrations and represent the coefficients of b and c by

$$6 = \frac{4!}{2!2!}, \quad 3 = \frac{4!}{2!2!2!} \tag{14}$$

The Green's function can then be rewritten in the form

$$G^{(n)}(x_1, x_2, ..., x_n) = (i\lambda)^p AB, \tag{15}$$

where

$$
\begin{aligned}
A &\equiv \frac{(4!)^{p_2}(4!)^{p_3}}{(4!)^{(p_1+p_2+p_3)}(2!)^{p_2}(2!)^{p_2}(2!)^{p_3}(2!)^{p_3}(2!)^{p_3}p_1!p_2!p_3!}, && (16)\\
B &\equiv \langle 0|T[\phi(x_1)...\phi(x_n)a^{p_1}b^{p_2}c^{p_3}]|0\rangle && (17)
\end{aligned}
$$

We note that the b associated with p_2 contains one bubble diagram while the c associated with p_3, contains two, a double bubble $\bigcirc\!\bigcirc$. Hence if we let the β be the number of lines that connect a vertex to itself, then

$$\beta = p_2 + 2p_3. \tag{18}$$

Moreover, these bubbles can be factored out of the T-product in B such that the T–operator does not act on them:

$$B = \langle 0|T[\phi(x_1)...\phi(x_n)(N(\phi^4))^{p_1}(N(\phi^2))^{p_2}]|0\rangle \dot{\Delta}^{p_2}\dot{\Delta}^{2p_3}, \tag{19}$$

where the double bubbles (as disconnected pieces) are vacuum subdiagrams. We also note that p_2 and p_3 simply coincide with the corresponding s and d in the introduction.

The corresponding coefficient A is interpreted as

$$A = \left[\frac{1}{(4!)^{p_1}(2!)^{p_2}p_1!p_2!}\right]\left[\frac{1}{2^{\beta}(2!)^{p_3}p_3!}\right]. \tag{20}$$

In this formula, $p_1!$ and $p_2!$ are respective numbers of permutations of a and b vertices, similar to $p!$ in (8). The 4! (powered p_1) and 2! (powered p_2) are symmetry factors (the number of permutations of identical interaction-fields) respectively associated with a and b, similar to 4! in (7). In total, we obtain the factor $p_1!p_2!(4!)^{p_1}(2!)^{p_2}$, which is deduced as the first factor in (20). This factor can be simplified if we use the T-product expansion for B. The second factor, associated with the bubbles subdiagrams, is unchanged under T-product: $p_3!$ is the number of permutations of c vertices; 2! (powered p_3) is the number of permutations of the two single bubbles of any c vertex; β was described above.

Next, to contract B under the T-product, we refer to Eq. (4.45) in [5]. The number of different contractions that give the same expression is the product of four types of factors. First, we have $p_1!p_2!$ interchanges of p_1 a and p_2 b vertices. Second, for the placement of contractions into a vertex, we have 4! for a and 2! for b vertices and therefore $(4!)^{p_1}(2!)^{p_2}$ for p_1 a and p_2 b vertices (we note that there is no self-contraction for each vertex). Third, we have $1/\prod_{n=2,3...}(n!)^{\alpha_n}$ interchanges of vertex-vertex contractions, where n is the number of contractions and α_n is the number of vertex pairs with n contractions. Finally, if we let g' be the number of interchanges of a and b vertices that do not change the diagram topologically, then the factor $1/g'$ should be multiplied to the result. In summary, the total factor contributing to one diagram is

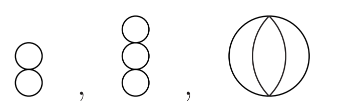
$$\frac{p_1!p_2!(4!)^{p_1}(2!)^{p_2}}{g'\prod_n (n!)^{\alpha_n}}A = \frac{1}{(g'p_3!)2^{\beta}(2!)^{p_3}\prod_n (n!)^{\alpha_n}} \tag{21}$$

Hence, the symmetry factor is given by

$$S = g2^{\beta}(2!)^{d}\prod_n (n!)^{\alpha_n}, \tag{22}$$

where $d = p_3$, and $g = g'p_3!$ has the same meaning as g'. We note that any vertex of a and b directly connected to the external points $x_1, x_2, ..., x_n$ is not subject to the interchanges defining g'. The examples in [6] and the followings examples demonstrate this.

The constructed diagram typically consists of connected pieces (subdiagrams), a piece connected to x_1, x_2,..., x_n and several pieces disconnected from all the external points, vacuum bubbles, in which the double bubble is one of the cases. We let V_c denote the connected piece and V_k denote the various possible disconnected pieces:

$V_k \in$  , $\cdots$

where $k = 1, 2, 3...$ We suppose that the diagram has n_k pieces of the form V_k for each k in addition to V_c. Let the value of g for the connected piece V_c and disconnected pieces V_k be g_c and g_k. It is easy to obtain $g = \prod_l n_l!(g_l)^{n_l}$, where $l = c,\ k$ and $n_c = 1$, $n_1 = p_3$. Here, $n_k!$ is the symmetry factor coming from interchanging the n_k copies of V_k. We can therefore rewrite (22) as

$$S = \prod_l n_l!(S_l)^{n_l} = S_c \times \prod_k n_k!(S_k)^{n_k}, \tag{23}$$

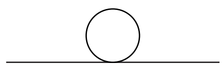
where $S_l = S_c,\ S_k$ is the symmetry factor of V_l having the same form as (22):

$$S_l = g_l 2^{\beta_l}(2!)^{d_l}\prod_n (n!)^{\alpha_n^l}, \tag{24}$$

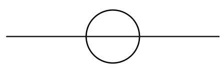
where the parameters indexed by l are those of V_l satisfying $d_{l=1} = 1$, $d_{l\neq 1} = 0$, $d = \sum_l n_l d_l$, $\beta = \sum_l n_l \beta_l$ and $\alpha_n = \sum_l n_l \alpha_n^l$. We note that there is an additional factor

2! associated with only double bubble. This contradicts formula (6), which is given in the literature.

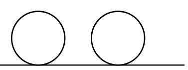
In calculating, we note that the symmetry factor of arbitrary diagrams is obtained from (22) or (23) while that of connected diagrams is given by (24). Because (22) and (24) have the same form, we can commonly use (22) for both the cases with the corresponding interpretation of the parameters. The symmetry factors of some two-point connected diagrams are



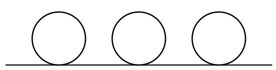
$S = 2 \ (g = 1, \beta = 1, d = 0, \alpha_n = 0)$



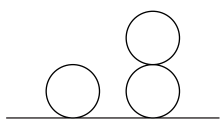
$S = 6 \ (g = 1, \beta = 0, d = 0, \alpha_3 = 1)$



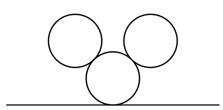
$S = 4 \ (g = 1, \beta = 2, d = 0, \alpha_n = 0)$



$S = 8 \ (g = 1, \beta = 3, d = 0, \alpha_n = 0)$

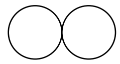


$S = 8 \ (g = 1, \beta = 2, d = 0, \alpha_2 = 1)$

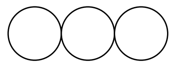


$S = 8 \ (g = 2, \beta = 2, d = 0, \alpha_n = 0)$

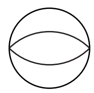
For some vacuum bubbles, we also have



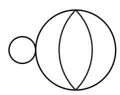
$S = 8\ (g = 1, \beta = 2, d = 1, \alpha_n = 0)$



$S = 16\ (g = 2, \beta = 2, d = 0, \alpha_2 = 1)$

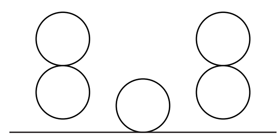


$S = 48\ (g = 2, \beta = 0, d = 0, \alpha_4 = 1)$



$S = 24\ (g = 2, \beta = 1, d = 0, \alpha_3 = 1)$

For general diagrams, we consider the example



$S = 2.2!(8)^2 = 256$, using (23).
Alternatively, $S = 256\ (g = 2, \beta = 5, d = 2, \alpha_n = 0)$, from (22).

More examples of symmetry factors are given in the following sections. In what follows, if some parameter has its trivial value (such us $g = 1$ or $\beta = 0$), then that parameter is not listed in parentheses. We next consider the case of complex scalar fields.

4 Symmetry factors in complex scalar theory

The interaction Lagrangian in the complex scalar theory is

$$\mathcal{L}^c_{int} = \frac{\rho}{4}(\varphi^*\varphi)^2 \tag{25}$$

Applying (5), we obtain

$$(\varphi^*\varphi)^2 \sim T[(\varphi^*\varphi)^2] = N[(\varphi^*\varphi)^2] + 4N(\varphi^*\varphi)\dot{\Delta} + 2\dot{\Delta}\dot{\Delta}, \tag{26}$$

where $\dot{\Delta}$ in this case denotes the bubble diagram *with arrow* . As before, we let a, b and c denote the corresponding terms. The pth-order Green's function is

$$G^{(p)}(x_1, x_2, ..., x_n) \quad = \quad (i\rho)^p A_c \langle 0|T[\varphi(x_1)...\varphi^*(x_n) a^{p_1} b^{p_2} c^{p_3}]|0\rangle, \tag{27}$$

where the integrations and summations are understood and

$$A_c \equiv \frac{1}{4^{p_1} 2^{p_3} p_1! p_2! p_3!}. \tag{28}$$

We note that the Green's function is nonzero only if the number of fields φ in (27) is equal to the number of their complex-conjugate fields φ^*.

Repeating the previous analysis, we obtain the contribution for one diagram

$$\frac{p_1! p_2! 4^{p_1}}{g' \prod_n (n!)^{\alpha_n}} A_c = \frac{1}{(g' p_3!) 2^{p_3} \prod_n (n!)^{\alpha_n}}. \tag{29}$$

Hence, the symmetry factor in the theory under consideration is given by

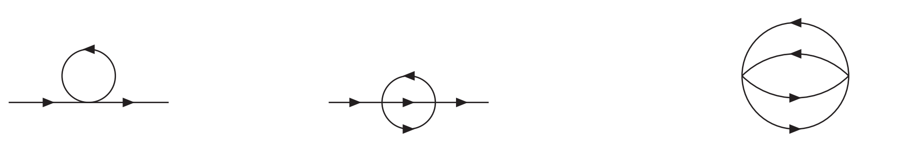
$$S = g 2^d \prod_n (n!)^{\alpha_n}, \tag{30}$$

where $d = p_3$ is the number of double bubbles, and $g = g' p_3!$ is the number of interchanges of interacting vertices leaving both the diagram and its charged scalar flows unchanged. As before, we can separate the symmetry factor into subfactors corresponding to connected and vacuum subdiagrams:

$$S = S_c \times S_v. \tag{31}$$

The symmetry factor for these subdiagrams has the same form as (30), where d is nonzero only if it is associated with a double bubble.

We emphasize that there is no factor 2^β in (30). We note that n is the number of identical lines connecting two separated vertices with *the same direction*. Formula (30) is simply a generalization of (22) discriminating between the scalar field directions. We illustrate this with the examples



(a) $S = 1$ $\qquad$ (b) $S = 2$ $(\alpha_2 = 1)$ $\qquad$ (c) $S = 8$ $(g = 2, \alpha_2 = 2)$

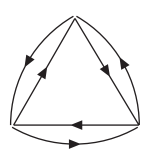
In the diagram (a) the symmetry factor is 1 because β is zero. In (b), we have only one set $n = 2$ and in (c), we have *two* sets with $n = 2$. We recall that in the real scalar theory, we have $n = 3$ and $n = 4$ for the corresponding diagrams. Many comparisons of symmetry factors of third-order diagrams in the real and complex scalar theories are given in the appendix.

It follows from Eq.(31) that the vacuum diagrams are factored order by perturbation theory order. Hence, the connected Green's functions, as in the literature, can be defined by the formula

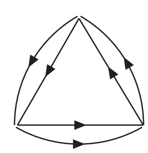
$$< 0|T[\varphi(x_1)\cdots\varphi(x_n)]|0>_c = \frac{\langle 0|T[\varphi(x_1)\cdots\varphi(x_n)\exp i\int d^4y\mathcal{L}_{int}]|0\rangle}{\langle 0|T\exp i\int d^4y\mathcal{L}_{int}|0\rangle}, \tag{32}$$

where the vacuum diagrams are contained in the denominator.

We next discuss some special properties of the complex theory. We consider two contributions with the symmetry factors



$S_1 = 6\ (g = 3!)$



$S_2 = 24\ (g = 3, \alpha_2 = 3)$

It is easy to verify that these contributions coincide because $\Delta(x,y) = \Delta(y,x)$ [11]. Hence, contributions of this type can be determined by only one diagram with the symmetry factor given by

$$S^{-1} = S_1^{-1} + S_2^{-1}, \tag{33}$$

and therefore $S = 24/5$.

We note the recently proposed hybrid inflationary scenario [12] in which there are two scalar fields ϕ and φ with the coupling

$$\frac{\lambda}{2}(\phi^2\varphi^2). \tag{34}$$

It is easily to verify that our formula is applicable to such interactions.

5 Conclusion

We have derived the symmetry factor for both the real and the complex scalar theories:

$$S = g 2^{\beta} 2^{d} \prod_{n} (n!)^{\alpha_n}, \tag{35}$$

where g is the number of interchanges of vertices leaving the diagram topologically unchanged, β is the number of lines connecting a vertex to itself (β is zero if the field is complex), d is the number of double bubbles, and α_n is the number of vertex pairs connected by n-identical lines. Our result revises the usual symmetry factor formula in the literature. Our result is easily generalized to higher-spin fields.

We have also showed that in the complex scalar theory, diagrams with different topologies can contribute the same. We also obtained the symmetry factor for contributions of such type.

It is easy to verify that our results are consistent with the symmetry factors in [4].

Our result explicitly shows that the vacuum diagrams, as expected, are factored order by perturbation theory order.

We recall that determining the symmetry factor is important because it not only is an important component of modern quantum field theory but also is used to calculate the effective potential in higher-dimensional theories and cosmological models.

Acknowledgments

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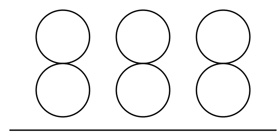
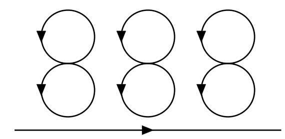
A Symmetry factors for three-order diagrams in the real and the complex scalar theories

Diagrams of the real scalar theory are given on the left below. The corresponding diagrams of the complex scalar theory are given on the right.



$S = 3072\ (g = 3!, \beta = 6, d = 3)$



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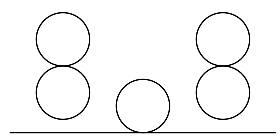
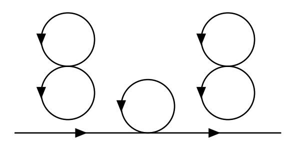


$S = 256\ (g = 2, \beta = 5, d = 2)$



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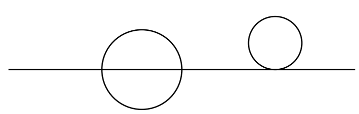
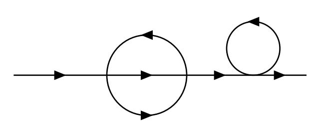


$S = 12\ (\beta = 1, \alpha_3 = 1)$



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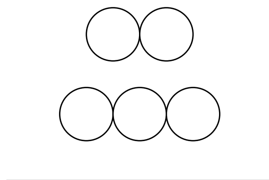

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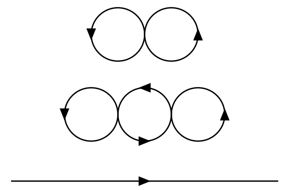
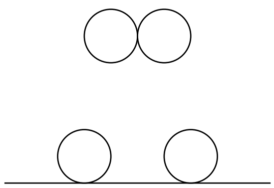
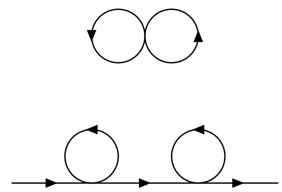


$S = 4\ (g = 2, d = 1)$



$S = 32\ (\beta = 4, d = 1)$



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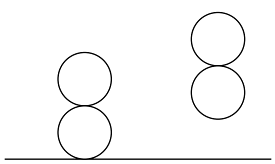

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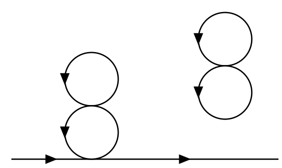

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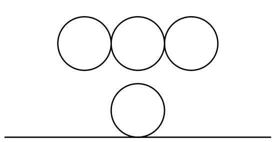

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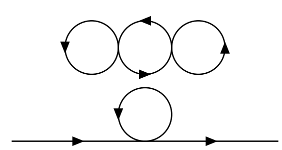

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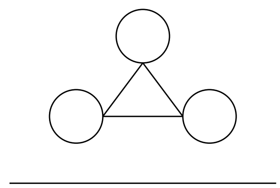
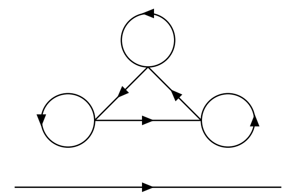


$S = 48\ (g = 3!, \beta = 3)$



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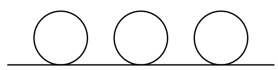
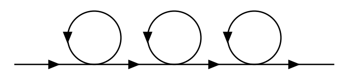
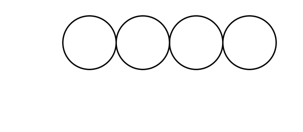


$S = 8\ (\beta = 3)$



$S = 1$



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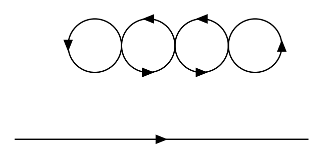

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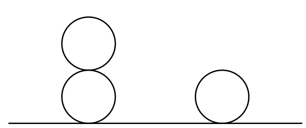

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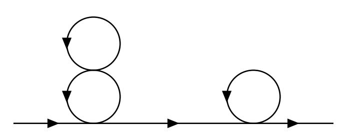

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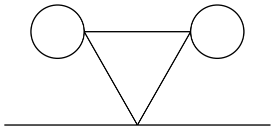

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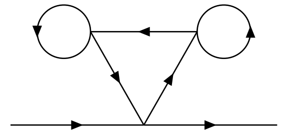

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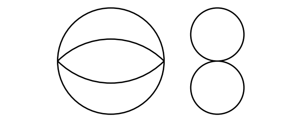

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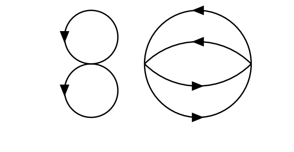

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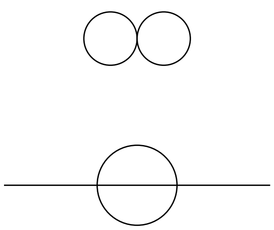
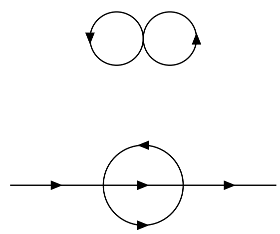


$S = 48 \; (\beta = 2, d = 1, \alpha_3 = 1)$



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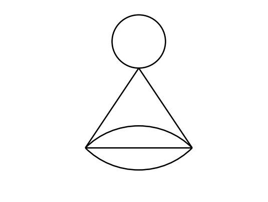

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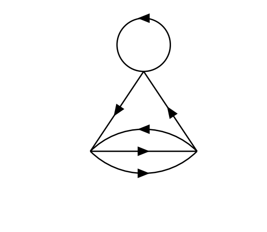

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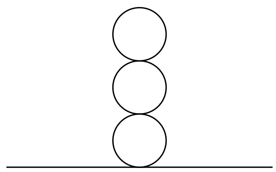

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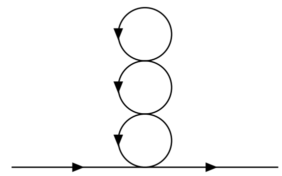

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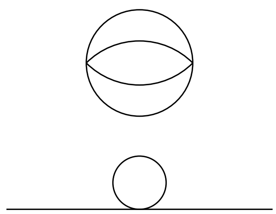

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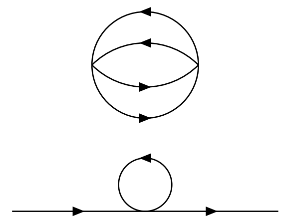

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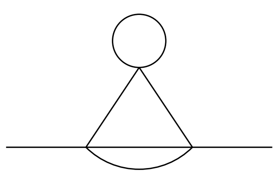

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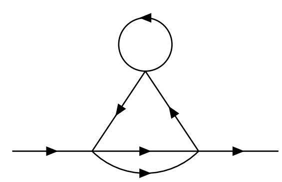

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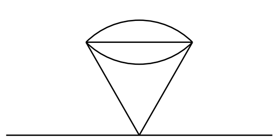

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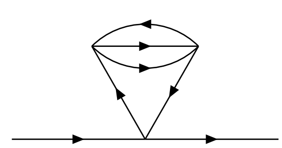

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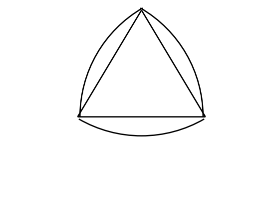

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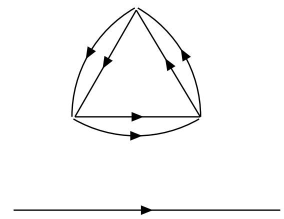

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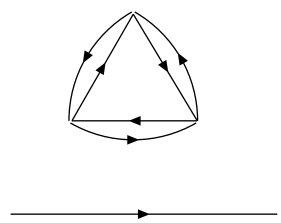

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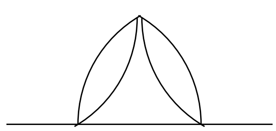

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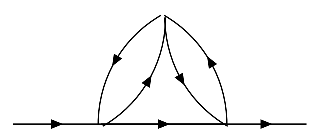
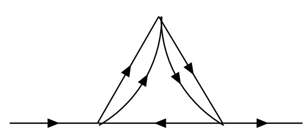


$S = 1$



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