Let K and L be symmetric PSD matrices of size N*N, with all entries in [0,1]. Let i be any number in 1...N and K', L' be two new symmetric PSD matrices, each with only row i and column i different from K and L.

I would like to obtain an upperbound to the quantity below:

$$
\begin{aligned}
&|[sum(K'.*L') - sum(K.*L)] \\
&- \frac{2}{N}[sum(K'L') - sum(KL)] \\
&+ \frac{1}{N^2}[sum(K')sum(L') - sum(K)sum(L)]|
\end{aligned}
$$

where $.*$ (MATLAB slang) means element-wise multiply.

Using simple triangular inequality and bounding the three square brackets respectively, I can bound the above with $12N - 13$. However, this is extremely loose and empirical experiments show that the constant coefficient should be much lower, probably closer to $1N$. Maybe there are linear algebra properties connecting those three forms of products that I don't know, would you please help me with some thoughts to get a better bound?