8.7.20

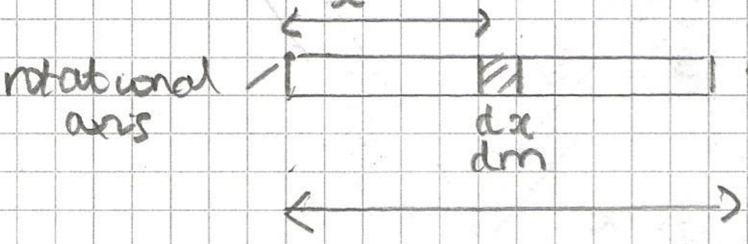
T-Shaped pendulum - Isaac Physics

Part A:

Parallel axis theorem:

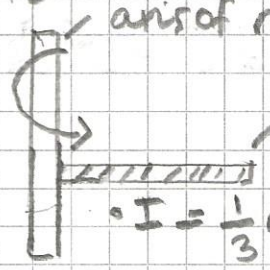
$$I = I_{CoM} + mL^2$$

consider the moment of inertia of the rod:

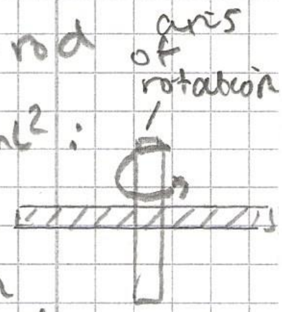


(no I_y) only rotates in one plane

• $I = \frac{1}{12}mL^2$ corresponds to:



• $I = \frac{1}{3}mL^2$:



$$dI = dm\,x^2$$

$$I = \int_{x=0}^{x=L} dI = \int_{x=0}^{x=L} dm\,x^2$$

introduce an integration variable to relate dm to dx:

① $dm = dx \times \frac{m}{l}$

sub ①

$$I = \int_{x=0}^{x=L} \frac{dx\,m}{L} x^2 = \frac{m}{L}\int_{x=0}^{x=L} x^2\,dx = \frac{m}{l}\left[\frac{x^3}{3}\right]_{x=0}^{x=L}$$

$$I = \frac{m}{L}\left[\frac{L^3}{3}\right]$$

$$I = \frac{1}{3}mL^2$$

CoM I of the horizontal rod; I of the vertical rod

$$I_{CoM} = \frac{1}{12}mL^2 + \frac{1}{3}mL^2$$

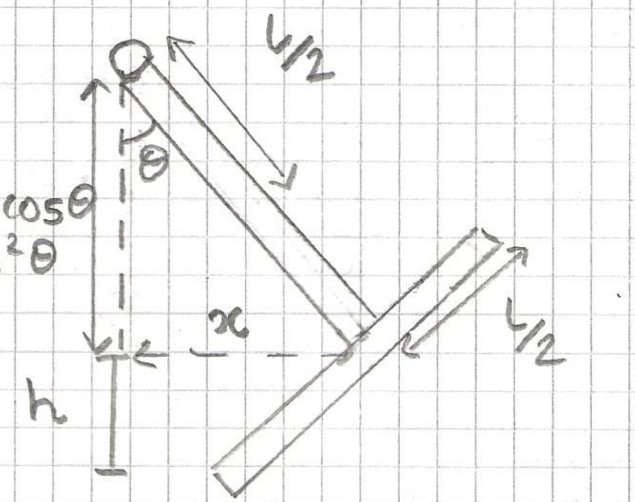
$$I = \frac{1}{12}mL^2 + \frac{1}{3}mL^2 + mL^2 = \frac{17}{12}mL^2$$

$$I = \frac{17}{12} \times 0.1 \times 0.31^2 = 0.0136\,kgm^2$$ ✓

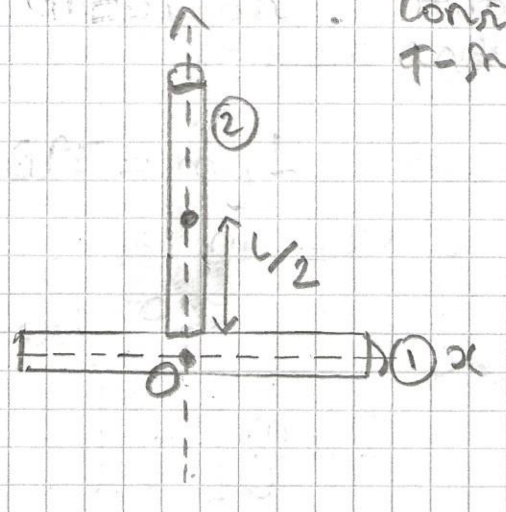
$\frac{1}{3}mL^2 > \frac{1}{12}mL^2$ because the moment of inertia increases as the rotation axis is moved further from the CoM. It is always minimised when the axis of rotation passes through the CoM.

Part B

$x^2 = L^2 - L^2\cos^2\theta$



thin rod ⇒ 1D



Consider the CoM of the whole T-shaped rod:

rod ①:

CoM at: (0,0) → 1D rod

rod ②:

CoM at: $\left(0, \frac{L}{2}\right)$

$\therefore x_{CoM} = 0$

$$Y_{CoM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{(m \times 0) + m\left(\frac{L}{2}\right)}{2m}$$

$$Y_{CoM} = \frac{L}{4} \Rightarrow \left(0, \frac{L}{4}\right)$$

from O.

$$h = \frac{L}{4} - \frac{L}{4}\cos\theta$$

$$GPE = 2mg\frac{L}{4}\left(1 - \cos\theta\right) = 2 \times 0.1 \times 9.81 \times \frac{0.31}{4}\left(1 - \cos 30\right)$$