This example is going to be somewhat trivial since I'm going to try and keep the numbers neat and tidy. Let

$$A = \begin{bmatrix} -2 & i \\ -i & -2 \end{bmatrix}$$

be our hermitian matrix. Then we are to compute $(AA^*)^{1/2} = \left(A^2\right)^{1/2}$. First, since A is hermitian we know it is diagonalizable via a unitary:

$$A = W\Lambda W^* = \begin{bmatrix} -i/\sqrt{2} & i/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} -3 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} i/\sqrt{2} & 1/\sqrt{2} \\ -i/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}.$$

This is just diagonalizing using eigenvectors and eigenvalues. For example, -3 is an eigenvalue and $\begin{bmatrix} -i/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}^T$ is a unit eigenvector. Now,

$$AA^* = A^2 = (W\Lambda W^*)(W\Lambda W^*) = W\Lambda^2 W^* = W \begin{bmatrix} 9 & 0 \\ 0 & 1 \end{bmatrix} W^*.$$

Then, as was defined in the PhysicsForums post,

$$(AA^*)^{1/2} = \left(A^2\right) = W \left(\Lambda^2\right)^{1/2} W^*$$

where

$$\left(\Lambda^2\right)^{1/2} = \begin{bmatrix} 9^{1/2} & 0 \\ 0 & 1^{1/2} \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}.$$

Remember, this is where we take the nonnegative square roots. Thus, putting it all together:

$$P = (AA^*)^{1/2} = \left(A^2\right) = W \left(\Lambda^2\right)^{1/2} W^* = \begin{bmatrix} -i/\sqrt{2} & i/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i/\sqrt{2} & 1/\sqrt{2} \\ -i/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 2 & -i \\ i & 2 \end{bmatrix}.$$

Now, we need the other half of our decomposition: the matrix U. But as was pointed out in the PhysicsForums post, when A is nonsingular this is pretty easy:

$$U = P^{-1}A = \frac{1}{3} \begin{bmatrix} 2 & i \\ -i & 2 \end{bmatrix} \begin{bmatrix} -2 & i \\ -i & -2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I.$$

Therefore,

$$A = \begin{bmatrix} -2 & i \\ -i & -2 \end{bmatrix} = \begin{bmatrix} 2 & -i \\ i & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = PU.$$

Again, this was a pretty trivial example, but hopefully you get the idea. However, it seems that when A is hermitian itself, the polar decomposition takes on a very simple form. I'll let you investigate why. :)