The Heisenberg Uncertainty Principle
A brief introduction and experimental verification for light

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Perhaps one of the most fundamental concepts of quantum mechanics is complementarity. "Bohr's principle of complementarity is the most revolutionary scientific concept of this century and the heart of his fifty-year search for the full significance of the quantum idea."[1] This keystone concept underpins much of the core elements of quantum theory, such as wave particle duality, as we can view an electron, for example, as a wave or a particle, but not both simultaneously. These two characteristics of the electron are mutually complementary – the closer we come to complete knowledge of either theories, the more complete the uncertainty of its counterpart is. A good analogy that demonstrates this concept is the well-known figure-ground vase **(Fig. 1)**. We can view this famous figure as a vase, or two people, but not both at the same time. The complication arises because it *is* both. In the same sense that an electron *is* both a wave and a particle in different situations, we cannot understand and observe this complementarity[2]. The basic understanding behind this report and indeed the beginnings of quantum mechanics is this inherent inability to explain and observe the existence of matter in these two defined states, but the understanding that it is in this mutually complementary state.

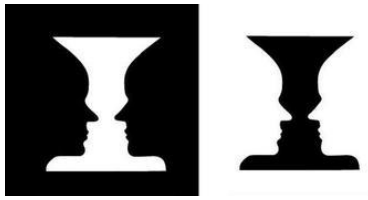


Fig. 1 – Rubin's Vase, or the figure-ground vase.

The Heisenberg uncertainty principle is a quantified case of this concept of complementarity. It is the mutual complementarity of the position, x, and momentum, p, of a particle, whereby the more complete knowledge we have of one of the quantities, the more complete uncertainty we have of the other. This relationship is defined as follows, where $\hbar = h/2\pi$:

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

This uncertainty relationship was first observed by Shull for neutrons in 1969[3], and it is widely considered that this uncertainty relationship will remain robust for all isolated matter. The most commonly observed case where this principle comes into play is with single slit diffraction, whereby a restricting slit is slowly closed, therefore reducing Δx, and this results in a momentum transfer to move the particles to the first order minimum in the diffraction pattern, therefore increasing Δp_x. This ensures that the above relationship is held for each particle.

With this basic understanding of complementarity in quantum mechanics and the uncertainty relationship above, I wanted to verify the relationship quantitatively for light. The apparatus used is outlined in **Fig. 2**. However, the particular relationship as shown above only applies to particular particles, or wave packets, considering the wave theory that leads to this phenomenon. An example of a group of particles that adhere to the above relationship are those following the Gaussian normal distribution. For this application, therefore, I had to derive the new uncertainty relationship relevant to photons. Starting with the de Broglie wavelength equation relevant to photons and the diffraction grating equation (whereby the velocity vector is calculated due to the momentum transfer through the angle required to move a photon to the first order minimum), the wave and particle theories of light can be combined to obtain this uncertainty relationship. This clearly provides evidence to support the complementarity of

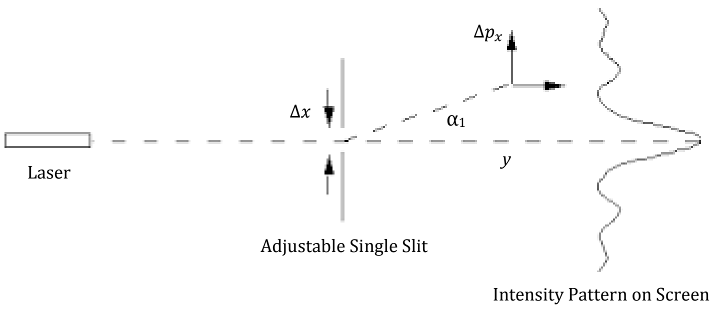


Fig. 2 – Apparatus – a laser with wavelength 632.8E-9m is diffracted by the slit.

[1] Physics Today, Issue 16 (Jan 1963), pg. 30.
[2] www.upscale.utoronto.ca/PVB/Harrison/Complementarity/CompCopen.html.
[3] Physical Review, Issue 179 (1969), pg. 752-754.

the two theories – this experiment demonstrates both the diffraction and particle nature of light, and the combination of the two effects is mathematically verified. We can only observe one effect happening at a given moment, but this proves that the two effects are happening simultaneously, which is quite remarkable if only as a thought experiment.

The full derivation from these two equations is as follows:

1. $p = \frac{h}{\lambda}$ (de Broglie) **2.** $sin\,\alpha_1 = \frac{n\lambda}{d}$ (diffraction grating)

Using trigonometry, and letting *n*=1, as first order minima are considered only:

3. $\Delta p_x = \frac{h}{\lambda} sin\,\alpha_1$ *and* **4.** $sin\,\alpha_1 = \frac{\lambda}{d}$

Combine to give:

$$\Delta p_x = \frac{h}{\lambda}\frac{\lambda}{d}$$

5. $\Delta p_x = \frac{h}{d}$

Also, considering Δx is effectively equal to *d*, as the uncertainty in position is equivalent to the size of the restricting slit:

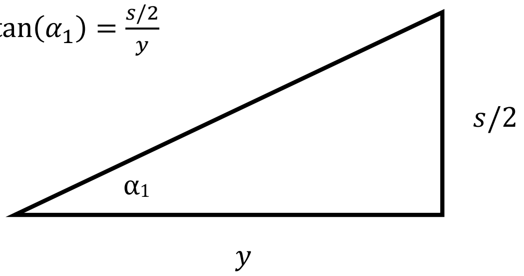
6. $\Delta p_x = \frac{h}{\Delta x}$

So therefore, for light:

7. $\Delta p_x \Delta x = h$

To quantitatively prove this relationship, however, α_1 will need to be found, and this can be calculated as follows, where s is the distance between first order minima:

$\arctan(\alpha_1) = \frac{s/2}{y}$



So, using the uncertainty relationship derived previously (3. and 7.):

$$\Delta p_x = \frac{h}{\lambda}\,\sin(\arctan\frac{d/2}{y})$$

8. $\frac{h}{\lambda}\sin(\arctan\frac{d/2}{y})\, d = h$

Dividing by h:

9. $\frac{d}{\lambda}\sin(\arctan\frac{d/2}{y}) = 1$

So to verify the uncertainty principle for light experimentally, data points well into the quantum regime of the experiment (i.e. they act according to the basic uncertainty principle whereby ΔP_x increases as Δx decreases), should adhere to the above equation and equal 1. Alternatively, the relationship can be verified by substituting values into the second above equation[4] and h should be the product.

With this prior mathematics and theory, I could begin the experiment. I varied the size of the restricting slit (which was a digital vernier caliper) from 0.00735m to 0.00005m, and recorded the distance between the two first order minima on a screen at a distance of y from the restricting slit, which was 2.911m. The results are summarized in **Fig. 3** below.

Δx (d)/m	s/m	α_1/rads	Δp/kgms^{-1} (E-31)
0.00735	0.01216	0.000826	0.9015
0.00635	0.01097	0.000794	1.0435
0.00535	0.00998	0.000795	1.2385
0.00435	0.00879	0.000763	1.5232
0.00335	0.00764	0.000737	1.9779
0.00235	0.00617	0.000656	2.8196
0.00135	0.00444	0.000531	4.9083
0.00100	0.00325	0.000386	6.6262
0.00080	0.00425	0.000593	8.2827
0.00070	0.00502	0.000742	9.4660
0.00060	0.00545	0.000833	11.043
0.00050	0.00736	0.001178	13.252
0.00040	0.00875	0.001434	16.565
0.00030	0.01071	0.001788	22.087
0.00020	0.01777	0.003018	33.131
0.00010	0.03692	0.006324	66.262
0.00005	0.06777	0.011631	132.52

Fig. 3 – Experimental data obtained.

The effect happening is demonstrated clearly in **Fig. 4** below.

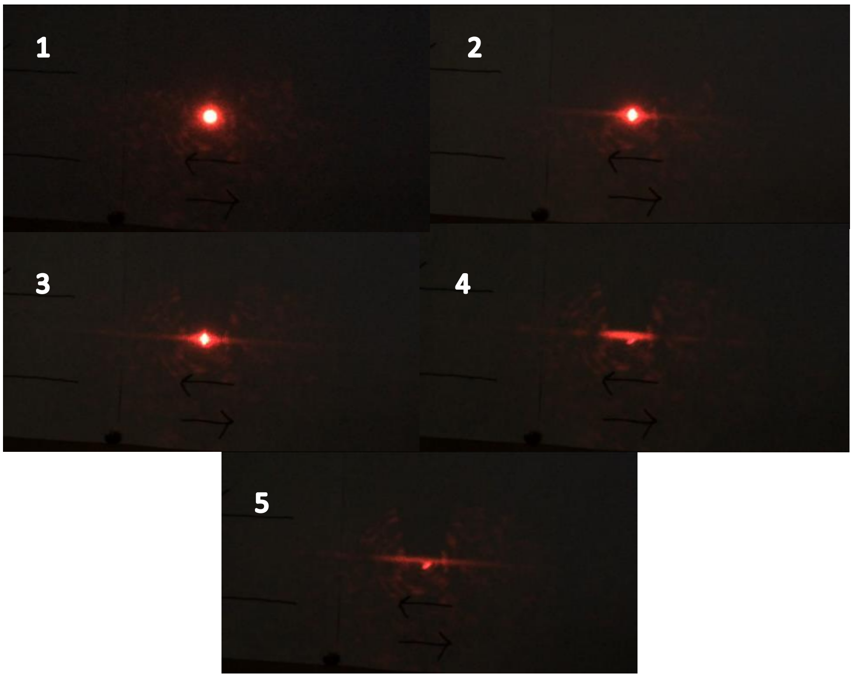


Fig. 4 – Photographs taken as the slit is restricted in size. The distance between minima decreases in the classical regime (1-3), before suddenly increasing in the quantum regime (4-5). This is explained later.

[4] PHYWE, *Diffraction at a slit and Heisenberg's uncertainty principle*, pg. 3.

It is important to note at this stage that the uncertainty relationship only comes into play at a restricting slit size of 0.001m, as above this value the slit size is not comparable to the distances at which quantum effects are observed, so Δx is effectively infinite in size at this macroscopic level[5]. This explains why the beam initially decreases in size, as the slit cuts the sides of the beam, before suddenly expanding, as the uncertainty relationship derived above takes effect. With this I could display the above data graphically, firstly plotting the distance between minima against Δx, to identify this boundary between the classical and quantum regimes, before plotting the final graph of Δp_x against Δx. This clearly shows the uncertainty principle, whereby as the slit closes, Δx decreases, so therefore Δp_x must increase. As p for a particular wavelength is always the same, the direction must change, therefore increasing Δp_x. These two graphs are shown below in **Fig. 5** and **Fig. 6**.

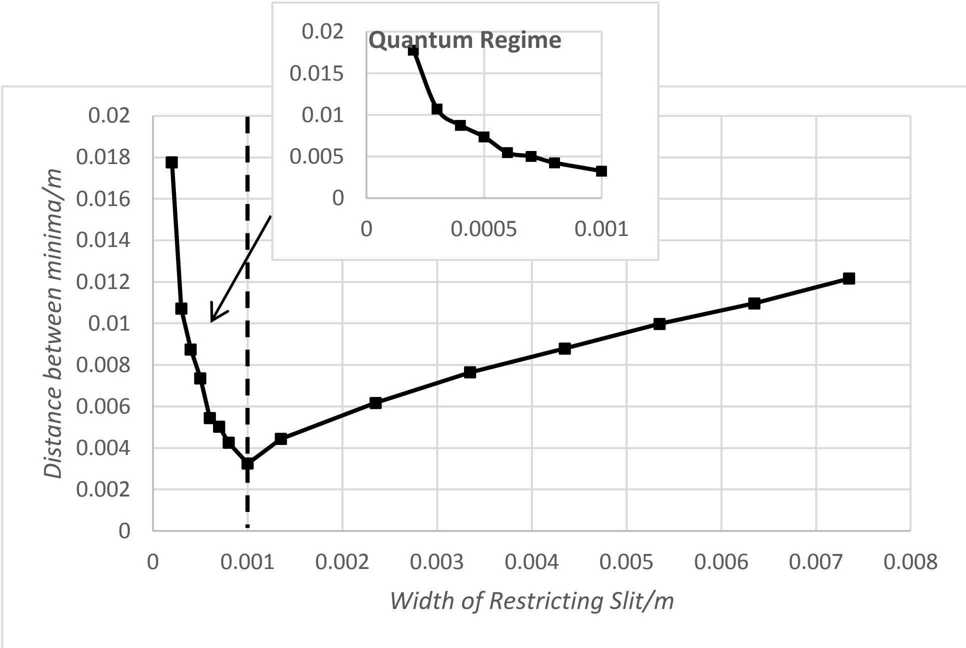


Fig. 5 – Initial graph to identify effect. The decreasing area from 0.001-0m (the quantum regime) is extrapolated above to show the uncertainty relationship.

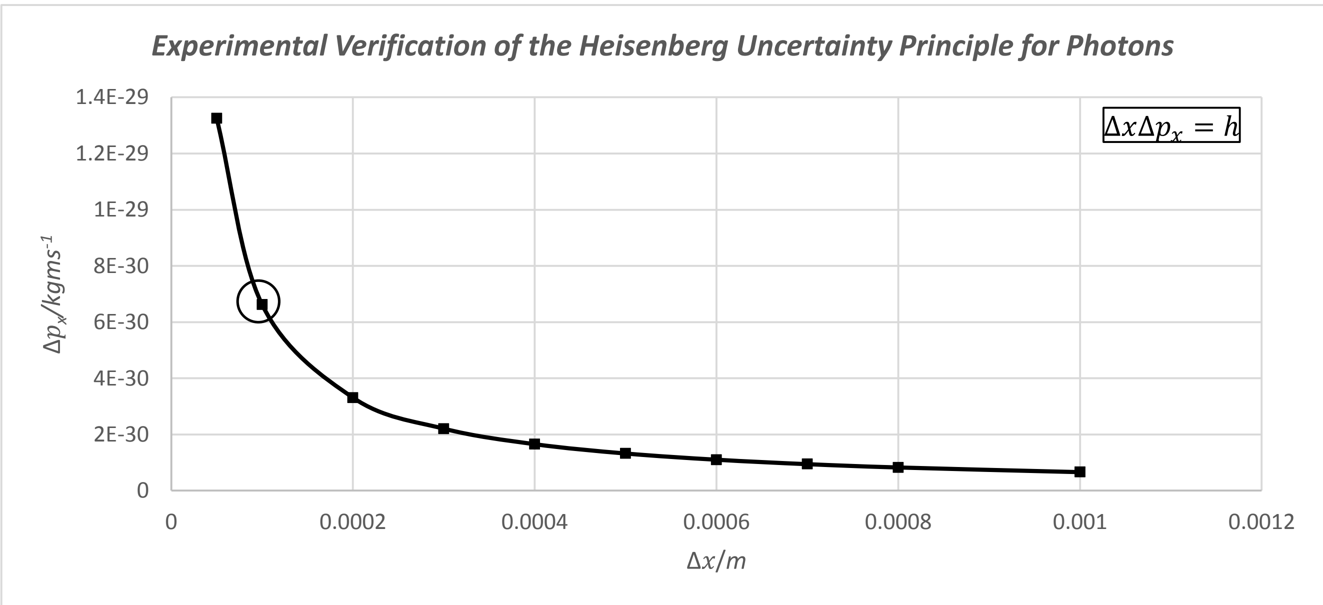


Fig. 6 – Final graph of Δp_x against Δx.

[5] www.upscale.utoronto.ca/PVB/Harrison/Complementarity/CompCopen.html.

With this final graph plotted, I could verify the uncertainty relationship defined earlier, and therefore Heisenberg's uncertainty principle for light.

By taking a point on the graph and multiplying the two components, *h* should be the result. For example, the second point on the graph (circled in **Fig. 6**), with the values substituted into the uncertainty relationship, results in 6.6262E-34, which is indeed *h* to 5 significant figures. This is a quite remarkable level of accuracy considering the relatively imprecise nature of the school laboratory equipment, and quite definitively confirms that light does indeed obey this core principle of quantum mechanics.

Going back to the original concept if complementarity, this experiment clearly demonstrates and quantifies the idea. We can observe the diffraction pattern that occurs around the central maximum, and this is a clear indication and proof of the wave theory of light. However, this experiment not only identifies and demonstrates the particle nature of light, but also shows that the two permutations are coexistent. The mathematical derivation shown above of the uncertainty principle is founded on both theories, and its proved existence here shows the simultaneous state of both theories that we fundamentally fail to understand.

Finally, possible options for further experimentation might involve up scaling the size of the incident particle, perhaps to molecules such as fullerenes, to prove that this principle still holds with sufficiently isolated matter, as Olaf Nairz, Markus Arndt and Anton Zeilinger researched in 2002.

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