We have determined the thermal input to be 265W and the maximum allowable skin temperature to be 305K. So based on this, we have to determine at what temperature the system will no longer be able to cope with the thermal input. Our primary heat transfer methods will be conduction and convection. Since there is more than one method at work here, we will use the general form of the heat transfer equation:

$$\dot{Q} = UA\Delta T$$

To determine the overall heat transfer coefficient, we use the formula:

$$\frac{1}{UA} = \frac{1}{hA_1} + \frac{d}{kA_2}$$

First, we need to determine the convection coefficient. To do that, we have to understand the characteristics of the fluid flow through the tubing. In the shirt, the flow is split between four 5' (1.5m), 3/8" OD nylon tubes. To characterize the flow, we need to know the Reynolds number:

$$Re = \frac{DV\rho}{\mu}$$

3/8" OD nylon tube most frequently has an ID of 0.275" (0.699 cm). To determine the velocity, we need to know the volumetric flow rate of the system (pump flow rate):

$$\dot{V}_{total} = 1.8\, gallons/min = 0.114\, L/s$$

Split evenly among the four tubes:

$$\dot{V}_H = \frac{0.114\, L/s}{4} = 0.0285\, L/s$$

Plugging it into the fluid velocity:

$$Q = VA$$

$$A = \frac{\pi d^2}{4} = \frac{\pi (0.699\, cm)^2}{4} = 0.384\, cm^2$$

$$0.0285\, L/s = (0.384\, cm^2)V$$

$$V = 0.742\, m/s$$

Substituting into the Reynolds equation:

$$Re = \frac{DV\rho}{\mu} = \frac{(0.699\, cm)(0.742\, m/s)(1000\, kg/m^3)}{1.78 \times 10^{-3}\, Pa \cdot s}$$

$$Re = 2900$$

Our flow is transitional, but it is close enough to turbulent that we can treat it as such. We also know our flow is fully developed because:

$$\frac{L}{D} = \frac{1500}{0.699} = 2146 \geq 50$$

Our tube is relatively smooth, so we can calculate the Nusselt number by the following equation:

$$Nu = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$$

The Darcy friction factor can be approximated by:

$$f = \frac{1}{[0.790 \ln(Re) - 1.64]^2} = \frac{1}{[0.790 \ln(2900) - 1.64]^2}$$

$$f = 0.046$$

And the Prandtl number can be found by (note that these values will change with temperature):

$$Pr = \frac{c_p \mu}{k} = \frac{(4204\, J/kg \cdot K)(1.78 \times 10^{-3}\, Pa \cdot s)}{0.596\, W/m \cdot K}$$

$$Pr = 12.56$$

Putting it all together:

$$Nu = \frac{(0.046/8)(2900 - 1000)12.56}{1 + 12.7(0.046/8)^{1/2}(12.56^{2/3} - 1)}$$

$$Nu = \frac{(0.00575)(1900)12.56}{1 + 12.7(0.076)(4.40)} = \frac{137.22}{5.25}$$

$$Nu = 26.14$$

Substituting for the convection coefficient:

$$Nu = \frac{hL}{k}$$

$$26.14 = \frac{h(1.5\ m)}{0.596\, W/m \cdot K} = 2.52h$$

$$h = \frac{26.14}{2.52} = 10.37\, W/m^2 \cdot K$$

Now to find the inner surface area of all one tube:

$$A_1 = \pi dL = \pi(0.699\ cm)(1.5\ m) = 0.033\ m^2$$

Substituting back in:

$$\frac{1}{UA} = \frac{1}{(10.37\, W/m^2 \cdot K)(0.033\ m^2)} + \frac{d}{kA_2}$$

For the conduction term, we already know the conduction coefficient and wall thickness:

$$k_{nylon} = 0.25\, W/m \cdot K$$

$$d = r_o - r_i = \frac{0.375''}{2} - \frac{0.275''}{2} = 0.05'' = 0.127\ cm$$

And now the conductive surface area for one tube:

$$A_2 = \pi DL = \pi(0.953\ cm)(1.5\ m) = 0.045\ m^2$$

Substituting terms:

$$\frac{1}{UA} = \frac{1}{(10.37)(0.033)} + \frac{0.127\ cm}{(0.25\, W/m \cdot K)(0.045\ m^2)}$$

$$\frac{1}{UA} = 2.922 + 0.113 = 3.035$$

$$UA = 0.33\, W/K$$

Solving for our heat transfer equation:

$$\dot{Q} = UA\Delta T$$

$$66.25W = (0.33\, W/K)\Delta T$$

$$\Delta T = \frac{66.25}{0.33} = 200.8\ K$$